Gravity and Circles Packet

1. (I) A horizontal force of 210 N is exerted on a 2.0-kg discus as it rotates uniformly in a horizontal circle (at arm's length) of radius 0.90 m. Calculate the speed of the discus.

The speed can be found from the centripetal force and centripetal acceleration.

$$F_R = ma_R = mv^2/r \quad \rightarrow \quad v = \sqrt{\frac{F_R r}{m}} = \sqrt{\frac{(210\ \text{N})(0.90\ \text{m})}{2.0\ \text{kg}}} = \boxed{9.7\,\text{m/s}}$$

2. (II) What is the maximum speed with which a 1050-kg car can round a turn of radius 77 m on a flat road if the coefficient of static friction between tires and road is 0.80? Is this result independent of the mass of the car?

A free-body diagram for the car at one instant of time is shown. In the diagram, the car is coming out of the paper at the reader, and the center of the circular path is to the right of the car, in the plane of the paper. If the car has its maximum speed, it would be on the verge of slipping, and the force of static friction would be at its maximum value. The vertical forces (gravity and normal force) are of the same magnitude, because the car is not accelerating vertically. We assume that the force of friction is the force causing the circular motion.

$$F_R = F_{fr} \quad \rightarrow \quad mv^2/r = \mu_s F_N = \mu_s mg \quad \rightarrow \quad v = \sqrt{\mu_s rg} = \sqrt{(0.80)(77\ \text{m})(9.8\,\text{m/s}^2)} = \boxed{25\,\text{m/s}}$$

Notice that the result is independent of the car's mass.

3. (II) A device for training astronauts and jet fighter pilots is designed to rotate a trainee in a horizontal circle of radius 12.0 m. If the force felt by the trainee on her back is 7.85 times her own weight, how fast is she rotating? Express your answer in both m/s and rev/s.

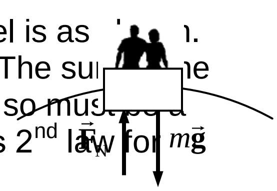
Since the motion is all in a horizontal circle, gravity has no influence on the analysis. Set the general expression for centripetal force equal to the stated force in the problem.

$$F_R = mv^2/r = 7.85W = 7.85mg \quad \rightarrow \quad v = \sqrt{7.85rg} = \sqrt{7.85(12.0\text{m})(9.8\,\text{m/s}^2)} = \boxed{30.4\,\text{m/s}}$$

$$(30.4\,\text{m/s})\left(\frac{1\ \text{rev}}{2\pi(12.0\ \text{m})}\right) = \boxed{0.403\,\text{rev/s}}$$

4. (II) How many revolutions per minute would a 15-m-diameter Ferris wheel need to make for the passengers to feel "weightless" at the topmost point?

The free-body diagram for passengers at the top of a Ferris wheel is as shown. F_N is the normal force of the seat pushing up on the passenger. The sum of the forces on the passenger is producing the centripetal motion, and so must be a centripetal force. Call the downward direction positive. Newton's 2nd law for the passenger is:



$$\sum F_R = mg - F_N = ma = mv^2/r$$

Since the passenger is to feel "weightless", they must lose contact with their seat, and so the normal force will be 0.

$$mg = mv^2/r \quad \rightarrow \quad v = \sqrt{gr} = \sqrt{\left(9.8\,\mathrm{m/s^2}\right)\left(7.5\,\mathrm{m}\right)} = 8.6\,\mathrm{m/s}$$

$$\left(8.6\frac{\mathrm{m}}{\mathrm{s}}\right)\left(\frac{1\text{ rev}}{2\pi\left(7.5\text{ m}\right)}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{11\text{ rpm}}$$

5. (I) Calculate the force of Earth's gravity on a spacecraft 12,800 km (2 Earth radii) above the Earth's surface if its mass is 1350 kg.

The spacecraft is three times as far from the Earth's center as when at the surface of the Earth. Therefore, since the force as gravity decreases as the square of the distance, the force of gravity on the spacecraft will be one-ninth of its weight at the Earth's surface.

$$F_G = \tfrac{1}{9}mg_{\substack{\text{Earth's}\\ \text{surface}}} = \frac{\left(1350\text{ kg}\right)\left(9.80\,\mathrm{m/s^2}\right)}{9} = \boxed{1.47\times10^3\,\mathrm{N}}$$

This could also have been found using Newton's law of Universal Gravitation.

6. (II) Calculate the acceleration due to gravity on the Moon. The Moon's radius is 1.74×10^6 m and its mass is 7.35×10^{22} kg.

The force of gravity on an object at the surface of a planet is given by Newton's law of Universal Gravitation, using the mass and radius of the planet. If that is the only force on an object, then the acceleration of a freely-falling object is acceleration due to gravity.

$$F_G = G\frac{M_{\text{Moon}}m}{r_{\text{Moon}}^2} = mg_{\text{Moon}} \quad \rightarrow$$

$$g_{\text{Moon}} = G\frac{M_{\text{Moon}}}{r_{\text{Moon}}^2} = \left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\frac{\left(7.35\times10^{22}\,\mathrm{kg}\right)}{\left(1.74\times10^6\,\mathrm{m}\right)^2} = \boxed{1.62\,\mathrm{m/s^2}}$$

7. (I) The space shuttle releases a satellite into a circular orbit 650 km above the Earth. How fast must the shuttle be moving (relative to Earth) when the release occurs?

The shuttle must be moving at "orbit speed" in order for the satellite to remain in the orbit when released. The speed of a satellite in circular orbit around the Earth is given by

$$v = \sqrt{G\frac{M_{\text{Earth}}}{r}} = \sqrt{G\frac{M_{\text{Earth}}}{\left(R_{\text{Earth}} + 650\text{ km}\right)}} = \sqrt{\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\frac{\left(5.97\times10^{24}\,\mathrm{kg}\right)}{\left(6.38\times10^6\,\mathrm{m} + 6.5\times10^5\,\mathrm{m}\right)}}$$

$$= \boxed{7.53\times10^3\text{ m/s}}$$

8. (II) During an *Apollo* lunar landing mission, the command module continued to orbit the Moon at an altitude of about 100 km. How long did it take to go around the Moon once?

The speed of an object in an orbit of radius *r* around the Moon is given by $v = \sqrt{G M_{\text{Moon}}/r}$, and is also given by $v = 2\pi r/T$, where *T* is the period of the object in orbit. Equate the two expressions for the speed and solve for *T*.

$$\sqrt{G M_{\text{Moon}}/r} = 2\pi r/T \quad \rightarrow$$

$$T = 2\pi\sqrt{\frac{r^3}{GM_{\text{Moon}}}} = 2\pi\sqrt{\frac{\left(R_{\text{Moon}} + 100\ \text{km}\right)^3}{GM_{\text{Moon}}}} = 2\pi\sqrt{\frac{\left(1.74\times10^6\,\text{m} + 1.0\times10^5\,\text{m}\right)^3}{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.35\times10^{22}\,\text{kg}\right)}}$$

$$= \boxed{7.08\times10^3\,\text{s}\left(\approx 2\ \text{h}\right)}$$

9. (II) Halley's comet orbits the Sun roughly once every 76 years. It comes very close to the surface of the Sun on its closest approach (Fig. 5–39). Estimate the greatest distance of the comet from the Sun. Is it still "in" the Solar System? What planet's orbit is nearest when it is out there? [*Hint*: The mean distance *s* in Kepler's third law is half the sum of the nearest and farthest distance from the Sun.]

Use Kepler's 3rd law to relate the orbits of Earth and Halley's comet around the Sun.

$$\left(r_{\text{Halley}}/r_{\text{Earth}}\right)^3 = \left(T_{\text{Halley}}/T_{\text{Earth}}\right)^2 \quad \rightarrow$$

$$r_{\text{Halley}} = r_{\text{Earth}}\left(T_{\text{Halley}}/T_{\text{Earth}}\right)^{2/3} = \left(150\times10^6\,\text{km}\right)\left(76\ \text{y}/1\ \text{y}\right)^{2/3} = \boxed{2690\times10^6\,\text{km}}$$

This value is half the sum of the nearest and farthest distances of Halley's comet from the Sun. Since the nearest distance is very close to the Sun, we will approximate that nearest distance as 0. Then the farthest distance is twice the value above, or $5380\times10^6\,\text{km}$. This distance approaches the mean orbit distance of Pluto, which is $5900\times10^6\,\text{km}$. $\boxed{\text{It is still in the Solar System, nearest to Pluto's orbit.}}$