Name: Key

Practice – 16.10 & 17.5 Superposition, Interference and Resonance

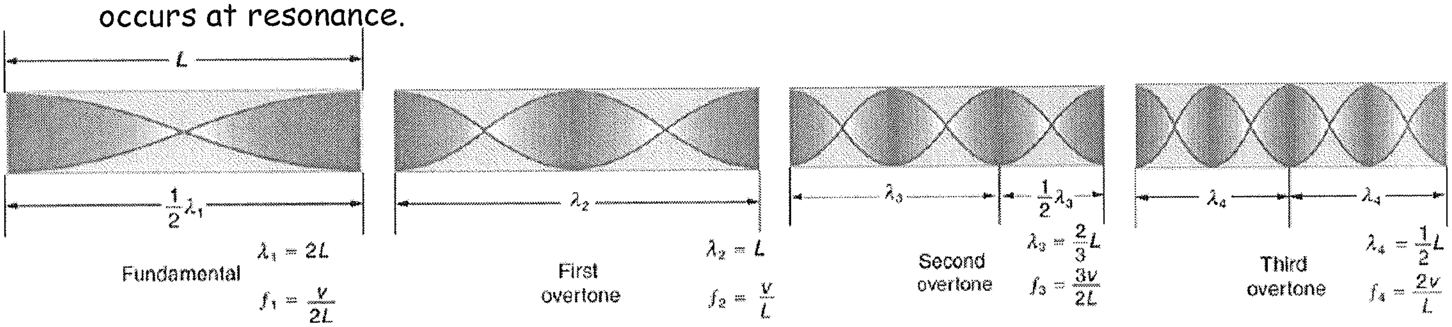
1. A "showy" custom-built car has two brass horns that are supposed to produce the same frequency but actually emit 263.8 and 264.5 Hz. What beat frequency is produced?

$f_B = |f_1 - f_2| = |263.8 - 264.5| = \boxed{0.7\text{ Hz}}$

2. A piano tuner hears a beat every 2.00 s when listening to a 264.0-Hz tuning fork and a single piano string. What are the two possible frequencies of the string?

$f_B = \frac{1}{T} = \frac{1}{2.00\text{ s}} = 0.500\text{ Hz} \Rightarrow f = \boxed{263.5\text{ Hz}, 264.5\text{ Hz}}$

3. Another type of tube is one that is open at both ends. Examples are some organ pipes, flutes, and oboes. The resonances of tubes open at both ends can be analyzed in a very similar fashion to those for tubes closed at one end. The air columns in tubes open at both ends have maximum air displacements at both ends. A standing wave occurs at resonance.



$v = \lambda f$

A. What is the fundamental frequency of a 0.672-m-long tube, open at both ends, on a day when the speed of sound is 344 m/s?

$\frac{1}{2}\lambda = L \Rightarrow \frac{1}{2}\left(\frac{v}{f}\right) = L \Rightarrow f_{1st} = \frac{v}{2L} = \frac{344\frac{m}{s}}{2(0.672\text{ m})} = \boxed{256\text{ Hz}}$

B. What is the frequency of its second harmonic?

$f_{2nd} = \frac{v}{L} = \frac{344\frac{m}{s}}{0.672\frac{m}{s}} = \boxed{512\text{ Hz}}$

4. How long must a flute be in order to have a fundamental frequency of 262 Hz (this frequency corresponds to middle C on the evenly tempered chromatic scale) on a day when air temperature is 20.0°C? It is open at both ends.

$$V = 331.3\sqrt{1+\frac{20.0°C}{273.15}} = 343\frac{m}{s}$$

$$f_{1st} = \frac{V}{2L} \Rightarrow L = \frac{V}{2f_{1st}} = \frac{343\frac{m}{s}}{2(262\,Hz)} = \boxed{0.655\,m}$$

5. A. Find the length of an organ pipe closed at one end that produces a fundamental frequency of 256 Hz when air temperature is 18.0°C.

$$\frac{1}{4}\lambda = L \Rightarrow \frac{1}{4}\left(\frac{V}{f}\right) = L \qquad V = 331.3\sqrt{1+\frac{18.0°C}{273.15}} = 342.04\frac{m}{s}$$

$$L = \frac{342.04\frac{m}{s}}{4(256\,Hz)} = \boxed{0.334\,m}$$

B. What is its fundamental frequency at 25.0°C?

$$V = 331.3\sqrt{1+\frac{25.0°C}{273.15}} = 346.13\frac{m}{s}$$

$$f = \frac{V}{4L} = \frac{346.13\frac{m}{s}}{4(0.3340\,m)} = \boxed{259\,Hz}$$

6. Students in a physics lab are asked to find the length of an air column in a tube closed at one end that has a fundamental frequency of 256 Hz. They hold the tube vertically and fill it with water to the top, then lower the water while a 256-Hz tuning fork is rung and listen for the first resonance.

A. What is the air temperature if this first resonance occurs for a length of 0.336 m?

$$V = 331.3\sqrt{1+\frac{T}{273.15}} \Rightarrow \left(\frac{V}{331.3}\right)^2 = 1 + \frac{T}{273.15} \Rightarrow T = 273.15\left(\left(\frac{V}{331.3}\right)^2 - 1\right)$$

$$f_{1st} = \frac{V}{4L} \Rightarrow V = 4Lf_{1st} \qquad T = 273.15\left(\left(\frac{4(0.336\,m)(256\,Hz)}{331.3}\right)^2 - 1\right) = \boxed{21.5°C}$$

B. At what length will they observe the second resonance (first overtone)?

$$L_1 = \frac{V}{4f} \qquad L_2 = \frac{3V}{4f} \Rightarrow L_2 = 3L_1 = 3(0.336\,m) = \boxed{1.01\,m}$$