Name: Key

Practice - 10.5 Angular Momentum and Its Conservation

1. A playground merry-go-round has a mass of 120 kg and a radius of 1.80 m and it is rotating with an angular velocity of 0.500 rev/s. What is its angular velocity after a 22.0-kg child gets onto it by grabbing its outer edge? The child is initially at rest.

$L_i = I_i \omega_i = \left(\frac{1}{2}MR^2\right)\omega_i = \frac{1}{2}(120\,kg)(1.80\,m)^2\left(0.500\,\frac{rev}{s}\right)\left(\frac{2\pi\,rad}{1\,rev}\right)$

$= 610.7\,\frac{kg\,m^2}{s}$

$L_f = I_f \omega_f \qquad L_i = L_f \Rightarrow 610.7 = I_f \omega_f$

$I_f = \frac{1}{2}(120\,kg)(1.80\,m)^2 + (22.0\,kg)(1.80\,m)^2 = 265.7\,kg\,m^2$

$\omega_f = \frac{L_i}{I_f} = \frac{610.7\,kg\,m^2/s}{265.7\,kg\,m^2} = \boxed{2.30\,\frac{rad}{s}}$

2. Ice Skater

A. Calculate the angular momentum of an ice skater spinning at 6.00 rev/s given his moment of inertia is 0.400 kg·m².

$L = I\omega = (0.400\,kg\,m^2)\left(6.00\,\frac{rev}{s}\right)\left(\frac{2\pi\,rad}{1\,rev}\right)$

$= \boxed{15.1\,\frac{kg\,m^2}{s}}$

B. He reduces his rate of spin (his angular velocity) by extending his arms and increasing his moment of inertia. Find the value of his moment of inertia if his angular velocity decreases to 1.25 rev/s.

$L_i = L_f \Rightarrow I_i\omega_i = I_f\omega_f \Rightarrow I_f = \frac{I_i\omega_i}{\omega_f}$

$I_f = (0.400\,kg\,m^2)\left(\frac{6.00\,rev/s}{1.25\,rev/s}\right) = \boxed{1.92\,kg\,m^2}$

C. Suppose instead he keeps his arms in and allows friction of the ice to slow him to 3.00 rev/s. What average torque was exerted if this takes 15.0 s?

$\tau = I\alpha$ and $\omega_f = \omega_i + \alpha t \Rightarrow \tau = I\left(\frac{\omega_f - \omega_i}{t}\right)$

$\Rightarrow \tau = (0.400\,kg\,m^2)\left(\frac{3.00\,\frac{rev}{s} - 6.00\,\frac{rev}{s}}{15.0\,s}\left(\frac{2\pi\,rad}{1\,rev}\right)\right) = \boxed{-0.503\,N\cdot m}$

3. What is the angular momentum of Earth rotating on its axis? $M_{Earth} = 5.97 \times 10^{24}$ kg and $R_{Earth} = 6371$ km. Assume the Earth is a solid uniform sphere.

$$L = I\omega = \left(\frac{2}{5}mR^2\right)\omega = \left(\frac{2}{5}\right)(5.97\times10^{24}\,kg)(6371\times10^3\,m)^2\left(\frac{2\pi\,rad}{24\,h}\right)\left(\frac{1\,h}{3600\,s}\right)$$

$$= \boxed{7.05\times10^{33}\,\frac{kg\,m^2}{s}}$$

4. What is the angular momentum of the Moon in its orbit around Earth? The orbital radius of the Moon is 384,399 km, the Moon's mass is 7.35×10^{22} kg and its orbital period is 27.321 days.

$$L = I\omega = mR^2\omega = (7.35\times10^{22}\,kg)(384{,}399\times10^3\,m)^2\left(\frac{2\pi\,rad}{27.321\,d}\right)\left(\frac{1\,d}{24\,h}\right)\left(\frac{1\,h}{3600\,s}\right)$$

$$= \boxed{2.89\times10^{34}\,\frac{kg\,m^2}{s}}$$