Name: Key

Practice – 10.4 Rotational Kinetic Energy: Work and Energy Revisited

1. What is the final velocity of a 1.0 kg hoop starting from rest that rolls without slipping down a hill 5.00 meters high? $I_{Hoop} = mR^2 \quad v = \omega R$

$$U = K_{trans} + K_{rot}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}mR^2\frac{v^2}{R^2}$$

$$= mv^2 \Rightarrow v = \sqrt{gh}$$

$$v = \sqrt{(9.80\tfrac{m}{s^2})(5.00m)} = \boxed{7.00\tfrac{m}{s}}$$

2. What is the final velocity of a 1.0 kg solid disk/cylinder starting from rest that rolls without slipping down a hill 5.00 meters high? $I_{cyl} = \frac{1}{2}mR^2$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right)$$

$$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$\cancel{m}gh = \frac{3}{4}\cancel{m}v^2 \Rightarrow v = \sqrt{\frac{4}{3}gh}$$

$$v = \sqrt{\frac{4}{3}(9.80\tfrac{m}{s^2})(5.00m)} = \boxed{8.08\tfrac{m}{s}}$$

3. Calculate the rotational kinetic energy of Earth on its axis. Assume the Earth is a uniform solid sphere of mass M = 5.97 x $10^{24}$ kg and a radius R = 6371 km.

$$I_{\text{solid sphere}} = \frac{2}{5}MR^2$$

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 = \frac{1}{5}MR^2\omega^2$$

$$= \frac{1}{5}\left(5.97\times10^{24}\,kg\right)\left(6371\times10^{3}\,m\right)^2\left[\frac{2\pi\,rad}{24h}\left(\frac{1h}{3600s}\right)\right]^2$$

$$= \boxed{2.56\times10^{29}\,J}$$

4. What is the rotational kinetic energy of Earth in its orbit around the Sun? M = 5.97 x $10^{24}$ kg and R = 150 million kilometers. $I = MR^2$

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(MR^2\right)\omega^2$$

$$= \frac{1}{2}\left(5.97\times10^{24}\,kg\right)\left(150\times10^{9}\,m\right)^2\left[\frac{2\pi}{365.25d}\left(\frac{1d}{24h}\right)\left(\frac{1h}{3600s}\right)\right]^2$$

$$= \boxed{2.66\times10^{33}\,J}$$

5. A ball with an initial velocity of 8.00 m/s rolls up a hill without slipping. Treating the ball as a spherical shell, calculate the vertical height it reaches.

$$I_{sph\,shell} = \frac{2}{3}MR^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{v^2}{R^2}\right)$$

$$= \frac{1}{2}mv^2 + \frac{1}{3}mv^2 = \frac{5}{6}mv^2$$

$$\cancel{m}gh = \frac{5}{6}\cancel{m}v^2 \Rightarrow h = \frac{5}{6}\frac{v^2}{g}$$

$$h = \frac{5}{6}\frac{\left(8.00\frac{m}{s}\right)^2}{9.80\frac{m}{s^2}} = \boxed{5.44m}$$