Name: Key

Practice - 10.3 Dynamics of Rotational Motion: Rotational Inertia

1. Calculate the rotational inertia of a solid sphere of mass M = 5.0 kg and a radius of R = 0.25 m.

$$I = \frac{2}{5}MR^2 = \frac{2}{5}(5.0\,kg)(0.25\,m)^2 = \boxed{0.13\ kg\,m^2}$$

0.125 $kg\,m^2$

2. Calculate the rotational inertia of a solid cylinder of mass M = 2.0 kg and a radius of R = 0.075 m about its central axis.

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2.0\,kg)(0.075\,m)^2 = \boxed{5.6\times10^{-3}\ kg\,m^2}$$

3. Suppose you exert a force of 180 N tangential to a 0.280-m-radius 75.0-kg grindstone (a solid disk).

A. What torque is exerted?

$$\tau = rF = (0.280\,m)(180\,N) = \boxed{50.4\ N\cdot m}$$

50.40 $N\cdot m$

B. What is the angular acceleration assuming negligible opposing friction?

$$\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I} = \frac{\tau}{\frac{1}{2}MR^2} = \frac{50.40\,N\cdot m}{\frac{1}{2}(75.0\,kg)(0.280\,m)^2} = \boxed{17.1\ \frac{rad}{s^2}}$$

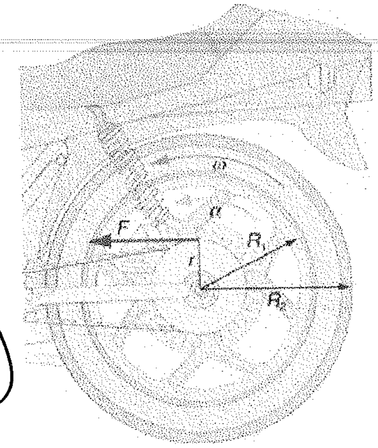
17.14 $\frac{rad}{s^2}$

C. What is the angular acceleration if there is an opposing frictional force of 20.0 N exerted 1.50 cm from the axis?

$$\alpha = \frac{\tau_{NET}}{I} = \frac{50.40 - (1.50\times10^{-3}\,m)(20.0\,N)}{\frac{1}{2}(75.0\,kg)(0.280\,m)^2} = \boxed{17.0\ \frac{rad}{s^2}}$$

17.04 $\frac{rad}{s^2}$

4. Consider a 12.0 kg motorcycle wheel to be approximately an annular ring with an inner radius of 0.280 m and an outer radius of 0.330 m. The motorcycle is on its center stand, so that the wheel can spin freely.



A. If the drive chain exerts a force of 2200 N at a radius of 5.00 cm, what is the angular acceleration of the wheel?

$$I = \frac{1}{2}m(R_1^2 + R_2^2) = \frac{1}{2}(12.0\,kg)\left((0.280\,m)^2 + (0.330\,m)^2\right)$$
$$= 1.124\,kg\,m^2$$

$$\alpha = \frac{\tau}{I} = \frac{(5.00\times10^{-2}\,m)(2200\,N)}{1.124\,kg\,m^2} = \boxed{97.9\,\frac{rad}{s^2}} \quad 97.88\,\frac{rad}{s^2}$$

B. What is the tangential acceleration of a point on the outer edge of the tire?

$$a_t = \alpha r = \left(97.88\,\frac{rad}{s^2}\right)(0.330\,m) = \boxed{32.3\,\frac{m}{s^2}}$$

C. How long, starting from rest, does it take to reach an angular velocity of 80.0 rad/s?

$$\omega = \cancel{\omega_0}^{0} + \alpha t \Rightarrow t = \frac{\omega}{\alpha} = \frac{80.0\,\frac{rad}{s}}{97.88\,\frac{rad}{s^2}} = \boxed{0.817\,s}$$

5. Zorch, an archenemy of Superman, decides to slow Earth's rotation once per 28.0 h by exerting an opposing force at and parallel to the equator. Superman is not immediately concerned, because he knows Zorch can only exert a force of 4.00×10^7 N (a little greater than a Saturn V rocket's thrust). How long must Zorch push with this force to accomplish his goal? (This period gives Superman time to devote to other villains.) Assume the Earth is a uniform solid sphere of mass $M = 5.97 \times 10^{24}$ kg and a radius $R = 6371$ km.

$$\omega_0 = \frac{2\pi \text{ rad}}{24\text{h}\left(\frac{3600\text{s}}{1\text{h}}\right)} = 7.272 \times 10^{-5} \frac{\text{rad}}{\text{s}}$$

$$\omega_f = \frac{2\pi \text{ rad}}{28\text{h}\left(\frac{3600\text{s}}{1\text{h}}\right)} = 6.233 \times 10^{-5} \frac{\text{rad}}{\text{s}}$$

$$\alpha = \frac{\tau}{I} = \frac{rF}{\frac{2}{5}MR^2} = \frac{(6371 \times 10^3\text{m})(4.00 \times 10^7\text{N})}{\frac{2}{5}(5.97 \times 10^{24}\text{kg})(6371 \times 10^3\text{m})^2} = 2.629 \times 10^{-24} \frac{\text{rad}}{\text{s}^2}$$

$$\omega_f = \omega_0 + \alpha t \Rightarrow t = \frac{\omega_f - \omega_0}{\alpha} = \frac{6.233 \times 10^{-5} - 7.272 \times 10^{-5} \frac{\text{rad}}{\text{s}}}{-2.629 \times 10^{-24} \frac{\text{rad}}{\text{s}^2}}$$

$$= \boxed{3.95 \times 10^{18}\text{ s}} \left(\frac{1\text{y}}{3.16 \times 10^7\text{s}}\right)$$

$$= \boxed{1.25 \times 10^{11}\text{ y}}$$