Physics 200 Name: Key

Notes - 10.3 Dynamics of Rotational Motion: Rotational Inertia

Demonstrate

1. A door opens more slowly if you push it closer to its hinges. The door will also open more slowly if it is more massive. From a torque and angular acceleration standpoint, the greater the applied force and the farther it is applied from the pivot point (the rotational axis), the greater the angular acceleration. The angular acceleration is inversely proportional to mass. These relationships are analogous to the familiar relationships of force, mass, and acceleration.

2. Starting with Newton's 2nd Law, derive an expression for torque t in terms of mass m, lever arm r and angular acceleration α (and introduce I - "Rotational Inertia" or "moment of inertia")

$\tau = Fr$

$\tau = Fr = mar$

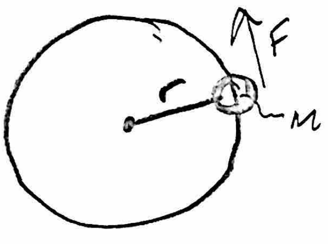
$F = ma$

$\tau = m(\alpha r)r$

$\tau = mr^2\alpha$

$I = mr^2$

$\tau = I\alpha$



3. Compare Newton's second law for linear motion and rotational motion.

$F = ma$ $\qquad F \leftrightarrow \tau$

$\tau = I\alpha$ $\qquad m \leftrightarrow I$

$\qquad a \leftrightarrow \alpha$

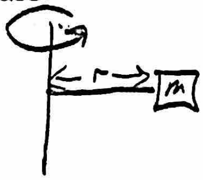
4. The two definitions of torque:

$\tau = Fr \qquad \tau = I\alpha$

5. Rotational Inertia (I) of Various Objects

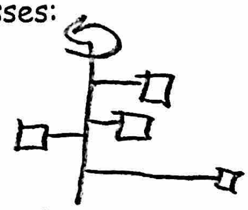
A. A single point mass:

$I = mr^2$



B. Multiple point masses:

$I = \Sigma m_i r_i^2$



C. Other shapes - see chart

	Object	Location of axis	Moment of inertia
(a)	Thin hoop, radius R	Through center	MR^2
(b)	Thin hoop, radius R width W	Through central diameter	$\frac{1}{2}MR^2 + \frac{1}{12}MW^2$
(c)	Solid cylinder, radius R	Through center	$\frac{1}{2}MR^2$
(d)	Hollow cylinder, inner radius R_1 outer radius R_2	Through center	$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	Uniform sphere, radius R	Through center	$\frac{2}{5}MR^2$
(f)	Long uniform rod, length L	Through center	$\frac{1}{12}ML^2$
(g)	Long uniform rod, length L	Through end	$\frac{1}{3}ML^2$
(h)	Rectangular thin plate, length L, width W	Through center	$\frac{1}{12}M(L^2 + W^2)$

Notes - 10.4 Rotational Kinetic Energy

1. Starting with the linear (or ~~tangential~~ Translational) kinetic energy formula, derive a formula for the rotational kinetic energy of a single mass m, with a velocity v, revolving around an axis at a radius r. The formula should be in terms of I and ω.

$KE_T = \frac{1}{2}mv^2$

$I = mr^2 \Rightarrow m = \frac{I}{r^2}$

$v = \omega r$

$KE = \frac{1}{2}\left(\frac{I}{r^2}\right)(\omega r)^2$

$KE_R = \frac{1}{2}I\omega^2$

2. Calculate the final speed of a solid cylinder that rolls down a 2.00-m-high incline. The cylinder starts from rest, has a mass of 0.750 kg, and has a radius of 4.00 cm.

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2$

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{4}\cancel{m}v^2 = \frac{3}{4}v^2$

$\frac{4}{3}gh = v^2 \Rightarrow v = \sqrt{\frac{4gh}{3}} = \sqrt{\frac{4(9.8 m/s^2)(2m)}{3}} = 5.11 m/s$

4. Calculate the final speed of a hoop of the same radius (4cm) that is allowed to roll down an incline of the same height (2m)

$\cancel{m}gh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}(\cancel{m}r^2)\left(\frac{v}{r}\right)^2$

$gh = \frac{1}{2}v^2 + \frac{1}{2}v^2 = v^2$

$v = \sqrt{gh} = \sqrt{(9.8 m/s^2)(2m)} = 4.43 m/s$

5. Compare the speeds of thin hoops and solid cylinders, in general, after rolling down ramps (assuming the objects' radii and the ramp heights are identical, and that there is no friction).

$\frac{V_{cyl}}{V_{Hoop}} = \frac{\sqrt{\frac{4\cancel{gh}}{3}}}{\sqrt{\cancel{gh}}}$ $\qquad \frac{V_{cyl}}{V_{Hoop}} = \sqrt{\frac{4}{3}}$

$V_{cyl} = V_{Hoop}\sqrt{4/3}$