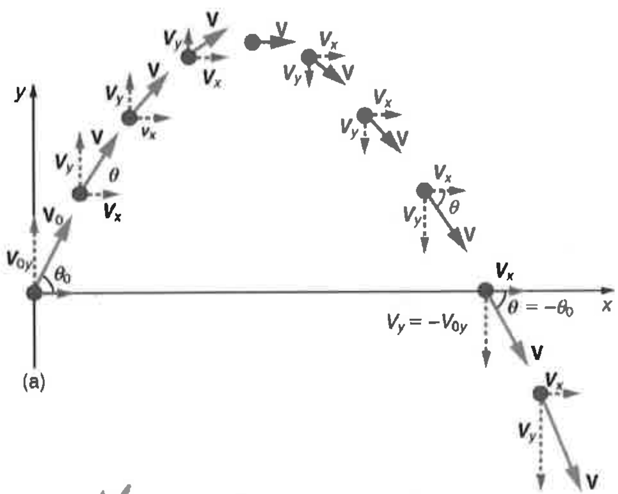
Physics 200 Name: ~~Answers~~ Answers

Notes and Practice – Currents and Projectiles

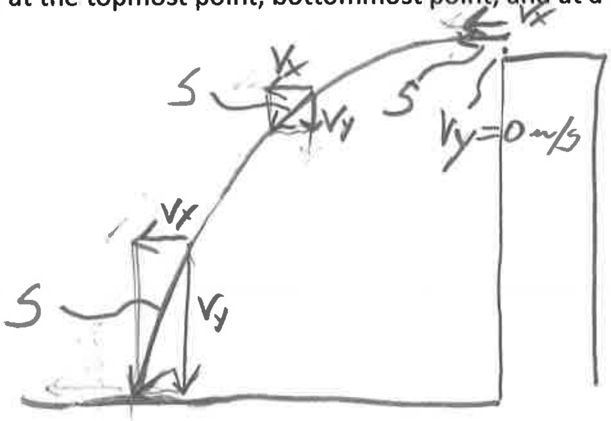
Projectiles:

1. The velocity of a launched projectile can be resolved into vertical (y) and horizontal (x) components. What happens to each of these components during the flight of the projectile? Why? Assume that there is no air resistance.

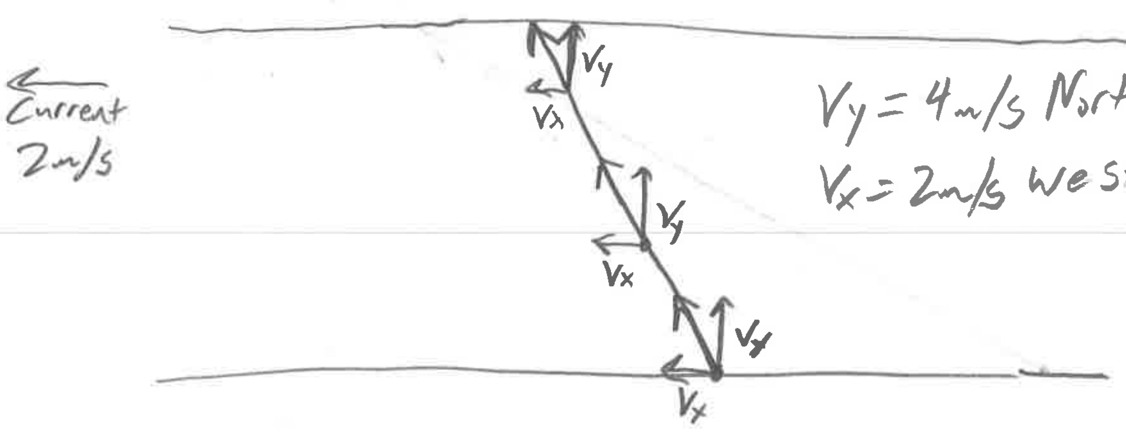


- V_y changes because gravity causes acceleration in y dimension.
- V_x is constant because there is nothing causing horizontal acceleration

2. A projectile is launched horizontally and to the left from the top of a tall building in the absence of air resistance. Sketch the path of the projectile as it falls to the ground. Use arrows to represent the object's speed, Vx, and Vy at the topmost point, bottommost point, and at a couple of other points in between.



3. Suppose a boat is launched directly northward across a river. The steering rudder is not adjusted once the trip is underway, and the boat's speed (relative to the water) is a steady 4m/s. The river's current has a westward velocity of 2m/s. What happens to the x and components of the boat's velocity as it crosses the river? Sketch a diagram showing the boat's path. For at least ~~three~~ two. points, sketch vectors representing the boat's speed and velocity components (Vx and Vy).

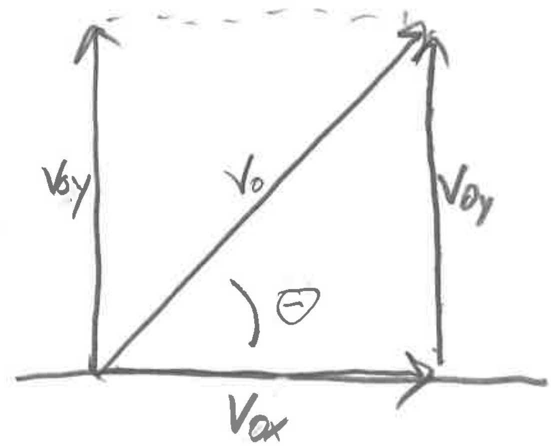


V_y = 4m/s North = motor

V_x = 2m/s west = current

A projectile is launched from ground level with an initial speed of V at an angle of θ above horizontal to the right. The projectile flies in the absence of air resistance until it returns to ground level.

4. Create a sketch showing the initial conditions in this problem. Show the initial velocity vector. Also resolve the initial velocity vector into X and Y components and sketch those components.



5. Use trig identities to provide the values of Vox and Voy.

Vox = $\cos\theta(V_0)$

Voy = $\sin\theta(V_0)$

6. Which component vector determines the time that the projectile remains in flight? Write a formula for time aloft.

V_{oy} determines time aloft

Time aloft $= \dfrac{2V_{oy}}{g} \Leftarrow$ Twice the ascent time $\left(\dfrac{V_{oy}}{g}\right)$

7. Write a formula for the maximum height reached by the projectile.

$$\Delta y = V_{oy}(t) + \tfrac{1}{2}(-g)t^2$$

$$\Delta y = V_{oy}\left(\frac{V_{oy}}{g}\right) + \tfrac{1}{2}(-g)\left(\frac{V_{oy}^2}{g^2}\right) \Rightarrow \Delta y = \frac{V_{oy}^2}{g} - \frac{V_{oy}^2}{2g} = \frac{V_{oy}^2}{2g}$$

8. Write a formula for the distance traveled by the projectile. This is known as the range formula.

$$\Delta x = V_x(\text{time aloft})$$

$$= \cos\theta(V_0)\left(\frac{2V_{oy}}{g}\right)$$

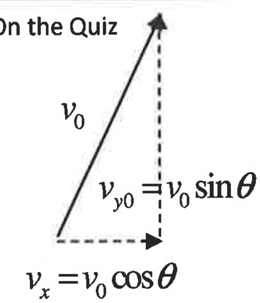
$$= \cos\theta V_0\left(\frac{2\sin\theta V_0}{g}\right)$$

$$= \frac{V_0^2\, 2\cos\theta\sin\theta}{g} = \frac{V_0^2 \sin 2\theta}{g}$$

$g = 9.8\ m/s^2$

Equations Provided On the Quiz

Resolving into x & y components:



Range formula:

$$Range = \frac{v_0^2 \sin 2\theta}{g}$$

Horizontal motion: $x = v_x t = v_0(\cos\theta)\,t$

Vertical Motion:

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2 = y_0 + v_0(\sin\theta)\,t - \frac{1}{2}gt^2$$

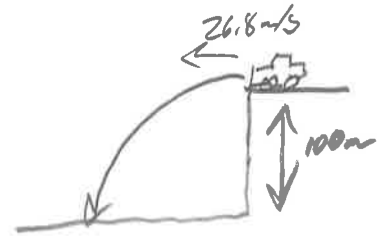
$$v_y = v_{y0} - gt = v_0 \sin\theta - gt$$

Projectile Practice Problems: Assume for all problems that there is no air resistance.

1. A car traveling at 60mph drives horizontally off of a cliff and falls to the ground 100m below.

a. Convert 60mph to m/s. $60\,mph\left(\frac{1\,m/s}{2.24\,mph}\right) = 26.8\,m/s$

b. How long does it take the car to reach the ground?



$V_{oy} = 0\,m/s$

$\Delta y = V_{oy}t + \frac{1}{2}(a)t^2$

$-100\,m = \frac{1}{2}(-9.8\,m/s^2)(t^2)$

$t^2 = \frac{-200\,m}{-9.8\,m/s^2}$

$t = 4.52\,s$

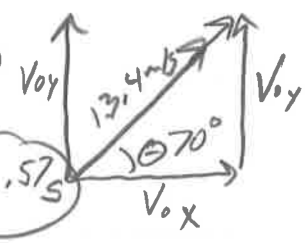
c. How far, horizontally, does the car fly through the air?

$\Delta x = \overline{V}_x(t) = 26.8\,m/s(4.52\,s) = 121\,m$

2. You throw a ball at a 70° angle with an initial speed of 30mph. The ball flies in an arc and lands on a shelf at the same height at which you released it.

a. Convert 30mph to m/s. $30\,mph\left(\frac{1\,m/s}{2.24\,mph}\right) = 13.4\,m/s$

b. How long will the ball remain aloft before hitting the shelf?



$V_{oy} = 13.4\,m/s\,(\sin\theta) = 13.4\,m/s\,(0.940)$

$V_{oy} = 12.6\,m/s$

$\text{Time aloft} = 2\left(\frac{V_{oy}}{g}\right) = 2\left(\frac{12.6\,m/s}{9.8\,m/s^2}\right) = 2.57\,s$

c. What is the distance between the point of release and the point of impact on the shelf?

$\Delta x = \overline{V}_x(\Delta t) = \cos(70°)(13.4\,m/s)(2.57\,s)$

$= 0.34(13.4\,m/s)(2.57\,s)$

$\Delta x = 11.8\,m$

d. What maximum height was reached by the ball?

$\text{ascent time} = \text{time aloft} \div 2 = \frac{2.57\,s}{2} = 1.29\,s$

$\Delta y = V_{oy}t + \frac{1}{2}at^2 = 12.6\,m/s(1.29\,s) + \frac{1}{2}(-9.8\,m/s^2)(1.29\,s)^2$

$= 16.3\,m - 8.15\,m$

$\Delta y = 8.15\,m$

3. You are trying to throw a ball through an open window that is 20m above the point at which you will release the ball and 5m in front of that release point. To minimize possible damage, you want the ball to enter the window at its apogee (max height). At what angle and with what initial speed should you release the ball?

1) Find V_{oy} $\quad V_y^2 = V_{oy}^2 + 2a(\Delta y)$

$0\,m/s = V_{oy}^2 + 2(-9.8\,m/s^2)(20\,m)$

$V_{oy} = 19.8\,m/s$

2) Find Ascent time $\Rightarrow \Delta t = \frac{V_{oy}}{9.8\,m/s^2} = \frac{19.8\,m/s}{9.8\,m/s^2} = 2.02\,s$

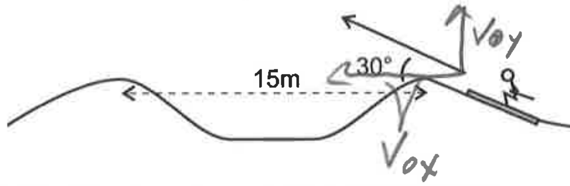
3) Find V_{ox} $\quad V_{ox} = \bar{V}_x = \frac{\Delta x}{\Delta t} = \frac{5\,m}{2.02\,s} = 2.47\,m/s$

4) Find V_0 $\quad V_0 = \sqrt{V_{xo} + V_{yo}} = \sqrt{(2.47\,m/s)^2 + (19.8\,m/s)^2}$

$V_0 = 20.0\,m/s$ = initial speed

5) Find θ $\quad \tan\theta = \frac{V_{oy}}{V_{oy}}$ $\quad \theta = \tan^{-1}\left(\frac{V_{oy}}{V_{ox}}\right) = \tan^{-1}\left(\frac{19.8\,m/s}{2.47\,m/s}\right) = 83^\circ$ above horizontal

4. A skier builds a jump and a landing area as shown on the diagram to the right. The takeoff point and the landing point are 15m apart and at equal elevations. The jump is inclined to horizontal at a 30 degree angle.



a. What speed does the skier need to attain in order to travel exactly 15 meters?

b. Given the initial speed from part a, what is the skier's maximum height, relative to the takeoff point?

c. Given the same initial speed, what is the skier's time aloft?

a) Using range formula... $\text{range} = \frac{V_0^2 \sin 2\theta}{g}$ $\quad 15\,m = \frac{V_0^2 \sin(60^\circ)}{9.8\,m/s^2}$

$\Rightarrow V_0^2 = \frac{15\,m(9.8\,m/s^2)}{\sin(60^\circ)} = 170\,m^2/s^2$ $\quad V_0 = 13\,m/s$

b) $V_{oy} = \sin(30^\circ)(13\,m/s) = 6.5\,m/s$ $\quad V_y^2 = V_{oy}^2 + 2a(\Delta y)$

$0\,m/s = (6.5\,m/s)^2 + 2(-9.8\,m/s^2)(\Delta y)$ $\quad \Delta y = 2.16\,m$

c) $\text{Time aloft} = 2\left(\frac{V_{oy}}{g}\right) = 2\left(\frac{6.5\,m/s}{9.8\,m/s^2}\right) = 1.33\,s$

or

$\Delta x = \Delta t(V_x) \Rightarrow 15\,m = \Delta t(\cos 30)(13\,m/s) \Rightarrow \Delta t = 1.33\,s$