Physics II
Water Rocketry

Water Rocket Project

Guidelines/Restrictions:

- Rockets must be propelled only by compressed air (90 psi gauge pressure) and tap water.
- Rockets must be compatible with class launcher (or your own launcher)
    - Nozzles must be compatible with 2-liter bottle caps.
    - Fins, etc. must not interfere with launcher "jaws."
- Rockets must be "made from scratch." Aside from basic construction materials (glue, tape...), parts must not be used for their intended purposes.
- Items that are allowed:
    - multi-stage rockets [I've never seen a successful one.]
    - rockets with very large (spliced together) compression chambers [very difficult to do].
    - parachutes

4th Quarter Grading (percentages approximate):

- (30% of quarter grade) Time aloft -- Cutoff for an "A" (93%) is 14 seconds. 20 seconds for an A+. 40 seconds for 100%. Extra credit for top 3 groups.
- (20%) Excel Rocket Flight Simulator Spreadsheet.
- (20%) Document your design process -- In a PowerPoint Presentation or a video, document your design process. Show the evolution that led to your final rocket design. Explain the changes that you made along the way, as well as why you made them. Share your discoveries and unanswered questions. Show your parachute ejection mechanism in detail. -- Keep it simple!! Use a digital camera (the class one or your own) take pictures of all of your modifications and rocket iterations.

Calculating Rocket Thrust -- The Basics:

Ft = Δmv. Ft is called "impulse." Impulse is the product of a force and the time over which the force is applied. When a force is applied to an object over a period of time, that object accelerates or decelerates. Thus the object's momentum changes. Impulse is equal to the change in the object's momentum.

By rearranging Ft = Δmv, we can get F= Δmv/t, or F = (Δm/t) (v). This final form is useful for rocket thrust. In a rocket, tiny particles are accelerated and ejected from the rocket. An "action" force accelerates those particles away from the rocket, and a "reaction" force accelerates the rocket in the other direction. In this final form of the impulse equation [$F = (\Delta m/t)\ (v_e)$], Δm represents the mass ejected from the rocket per unit of time, and v represents the velocity at which that mass is moving as it leaves the object. The more mass leaving per second, and the faster the mass is moving as it leaves, the greater the thrust generated. In the case of a water rocket, Δm/t is the mass of water being lost by the rocket per unit of time (in kg/s). The exit velocity (v_e) is the velocity (in m/s) at which water is exiting the rocket.

We have $\mathbf{F_{thrust} = (\Delta m/t) * v_e}$.

Calculating Water's Exit Velocity:

We can find v_e using Bernoulli's equation ($P_1 + 1/2\ \rho v_1^2 + \rho g h_1 = P_2 + 1/2\ \rho v_2^2 + \rho g h_2$). Remember that Bernoulli's equation applies to the pressure exerted on two different liquid surfaces, which are represented by the left and right sides of Bernoulli's equation.

First, let's assume that P_1 is the gauge pressure of the pressurized air in a rocket. If that's the case, we can simply leave out P_2 and use this gauge pressure for P_1. [Here's why we can do this: If we used actual pressures, on the left side of the equation, we would have $P_1 = P_{gauge} + P_{atmospheric}$, and on the right side of the equation, we would have $P_2 = P_{atmospheric}$. "$P_{atmospheric}$" values would be on both sides of the equation, so they would cancel one another out. We would be left with only P_{gauge} on the left side of the equation.] With P_2 gone, our equation now looks like ($P_1 + 1/2\ \rho v_1^2 + \rho g h_1 = 1/2\ \rho v_2^2 + \rho g h_2$).

Second, let's assume that the hole through which water is leaving is fairly small relative to the overall volume of water. In other words, there's enough of a constriction so that all of the water doesn't go whooshing out instantly. If this is the case, the velocity of the water leaving through the hole is much higher than the velocity of the surface of the water inside the rocket. In fact, that difference is so large that we will assume here that the water's upper surface has zero velocity. Setting $v_1 = 0$ allows us to get rid of some more of Bernoulli's equation. So now we have ($P_1 + \rho g h_1 = 1/2\ \rho v_2^2 + \rho g h_2$).

Third, a portion of Bernoulli's equation is concerned with pressure generated by the height of a column of fluid. In the case of our water rockets, the height of the water column in the rocket is very small, so it generates very little pressure. Furthermore, the pressure in the rocket is very high, causing the effects of this gravity-generated pressure to be even more negligable. So, let's leave out anything having to do with h_1 or h_2. This gives us our final useful formula, $P_1 = 1/2\ \rho v_2^2$, or $\mathbf{P_{gauge} = 1/2\ \rho v_e^2}$, where v_e is the exit velocity of the fluid in the rocket and ρ is the density of that fluid.

We can rearrange this to get $\mathbf{v_e = (2P_{gauge}/\rho_{water})^{1/2}}$.

Calculating Water Rocket Thrust -- Advanced:

Now we have two equations: one for thrust [$\mathbf{F_{thrust} = (\Delta m/t) * v_e}$] and one for exit velocity [$\mathbf{v_e = (2P_{gauge}/\rho_{water})^{1/2}}$]. These can be combined and simplified.

Calculating Rate of Mass Loss ($\Delta m/t$):

In order to calculate the thrust of a water rocket, we need to find a new way to write $\Delta m/t$, which can be thought of as mass of water ejected per unit of time. More precisely, we want to know how many kilograms of water are exiting the rocket each second. Since the rocket nozzle is a circle, we can visualize the water exiting the rocket as a cylinder. If we know how big that cylinder is and how fast it is shooting out of the nozzle, we can get an idea of how much mass is being lost every second.

In fact, we can easily calculate the cylinder's cross-sectional area. We can also easily calculate the speed with which the cylinder of water moves (using the v_e equation). The exit velocity of the cylinder of water, in meters per second, tells us the *length* of cylinder that exits the rocket per unit of time. If we know this length, and we also know the cross-sectional area of the cylinder, then we can calculate the *volume* of the cylinder that exits per unit of time. The formula for the volume of a cylinder is V= meters of length * area, so the volume of water exiting per second is: V/t = (meters of length/t) * area. The water's exit velocity, v_e tells us the meter of water that exit per second, so the volume of water exiting per second is $V/t = v_e*A$. The volume of water leaving the rocket per unit of time is a "change in volume," so this equation can be written as $\Delta V/t = v_e *A$.

Now we know the *volume* of water leaving the rocket per unit of time, but what we really want to know is the *mass* of water leaving the rocket per unit of time. Density = m/V, so $m=\rho V$. Therefore, if we take the equation from the previous paragraph (for *volume* of water leaving the rocket per unit of time), and we multipy this volume by the density of water, we get expressions for the *mass* of water lost per unit of time. $\rho(\Delta V/t) = \Delta m/t = \rho*v_e*A$. In summary, **rate of mass loss =** $\mathbf{\Delta m/t = \rho*v_e*A}$

Final Simplification of Water Rocket Thrust:

Finally, we can substitute $\mathbf{\Delta m/t = \rho*v_e*A}$ into the thrust equation $\mathbf{F_{thrust} = (\Delta m/t) * v_e}$. This gives us $F_{thrust} = (\rho*v_e*A)*v_e = \rho *v_e^2*A$.

From the previous page, $v_e = (2P_{gauge}/\rho)^{1/2}$, so $v_e^2 = 2P_{gauge}/\rho$. Thus, $F_{thrust} = \rho *v_e^2*A = \rho*(2P_{gauge}/\rho)*A = 2P_{gauge}A$.

So, for an air pressure-powered water rocket, $\mathbf{F_{thrust} = 2PA}$, where P = gauge pressure.

Water Rockets Provide 2X More "Kick" Than Potato Guns:

Note that water rocket thrust is twice as powerful as the thrust provided by a potato chunk that is being shot from a potato gun. In the case of a potato gun, the force of thrust is only PA.

The seeming contradiction stems from the fact that the potato accelerates from zero, finally achieving a maximum velocity as it leaves its barrel. The stream of water of a rocket, however, shoots from the rocket at a constant exit velocity, *seemingly* bypassing the act of acceleration. The potato's *average* velocity is half of its exit velocity, whereas the water's average velocity is equal to its exit velocity.

This means the water provides twice as much thrust, but it also means that the water leaves the rocket twice as fast. In each case, for equal ejected masses ejected by equal pressures, $Ft = \Delta mv$ is the same. The water provides a stronger force for a shorter time. The potato provides a weaker force for a longer time. In both cases, the change in momentum provided by these forces is the same.

Physics II Name: ______________________________

Water Rocket Calculations

A water rocket has been constructed using a 20-liter water cooler bottle for its compression chamber. The bottle has a mouth with an inner radius of 0.03 m. In preparation for the rocket's launch, the air in the rocket is pressurized to a gauge pressure of 40psi. Before pressurization, 60% of the bottle's volume was filled with water. The mass of the empty rocket (everything <u>except</u> water and added air) is 2.3 kg. The additional <u>air</u> added to the bottle during "inflation" adds another 0.04kg to the total mass of the rocket.

1. What is the initial exit velocity of the water that spews from this 20-liter bottle?

2. What is the initial rate at which mass is lost (in kg/s) from this bottle?

3. What initial thrust is produced by this expulsion of water?

4. What is the initial mass of water in the rocket?

5. What is the initial volume of air in the rocket?

6. What is the initial <u>total</u> mass of the rocket?

7. What is the initial total weight of the rocket?

8. What is the initial net force acting on the rocket?

9. What is the initial acceleration of the rocket?

<u>At these rates</u>, after 0.01 seconds have elapsed…

10. …what is the new volume of air in the rocket?

11. …assuming that $P_1V_1^{1.4} = P_2V_2^{1.4}$, what is the new air pressure inside the bottle?

Water Rocket Practice Problems:

A water rocket has been constructed using a 2-liter water bottle for its compression chamber. The bottle has a mouth with an inner radius of 0.01 m. In preparation for the rocket's launch, the air in the rocket is pressurized to a gauge pressure of 90psi. Before pressurization, 40% of the bottle's volume was filled with water. The mass of the empty rocket (everything <u>except</u> water and added air) is 0.15 kg. The additional <u>air</u> added to the bottle during "inflation" adds another 0.008kg to the total mass of the rocket.

1. What is the initial exit velocity of the water that spews from this 20-liter bottle?

2. What is the initial rate at which mass is lost (in kg/s) from this bottle?

3. What initial thrust is produced by this expulsion of water?

4. What is the initial mass of water in the rocket?

5. What is the initial volume of air in the rocket?

6. What is the initial <u>total</u> mass of the rocket?

7. What is the initial total weight of the rocket?

8. What is the initial net force acting on the rocket?

9. What is the initial acceleration of the rocket?

<u>At these rates</u>, after 0.01 seconds have elapsed…

10. …what is the new volume of air in the rocket?

11. …assuming that $P_1V_1^{1.4} = P_2V_2^{1.4}$, what is the new air pressure inside the bottle?