Physics 200 (Stapleton) Name: *Answers*

Notes: Circular Motion, Gravity, and Kepler's Laws

Circular Motion Formulas:	$a_{centripetal} = v^2/r$	$\sum F = mv^2/r$

1. [Horizontal circles] A 0.4kg ball on a string is swinging in horizontal circles at a constant speed of 3m/s. The radius of the orbit (i.e. the string length) is 0.5m. What is the tension in the string?

$$\text{Tension} = \sum F = \frac{mv^2}{r} = \frac{0.4kg\,(3m/s)^2}{0.5m} = 7.2N$$

2. [Vertical Circles] A car is approaching a "loop-the-loop" with a radius of 15m. What speed does the car need to maintain in order to maintain contact with the road, even when upside-down?

Neg net force

$\sum F$ is negative, because a is downward

$$\sum F = -\frac{mv^2}{r} = -w - \underbrace{F_N}_{\text{zero}}$$

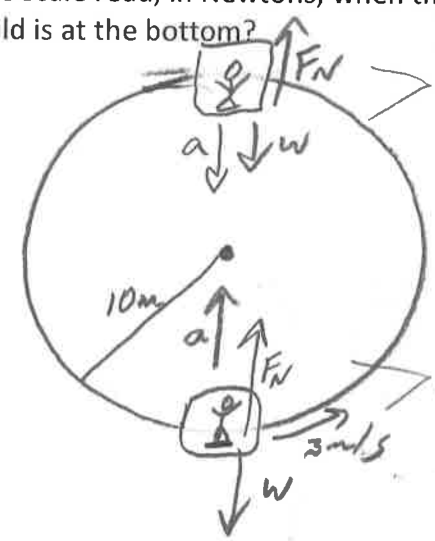
For contact with road, $F_N \geq 0$.

$$-\frac{\cancel{m}v^2}{r} = -w = -\cancel{m}g$$

$$\frac{v^2}{r} = g \qquad \frac{v^2}{15m} = 9.8$$

$$v = 12.1m/s$$

3. [Vertical Circles] A 20kg child is standing on a bathroom scale inside a Ferris Wheel that is rotating at a constant rate . The speed of the child is a constant 3m/s. If the radius (distance from child/scale to center) of the wheel is 10m, what does the scale read, in Newtons, when the child is at the top of the circle? What does it read when the child is at the bottom?



Neg net force

Pos net force

At Bottom $\sum F = \frac{mv^2}{r} = F_N - w$ (w = mg)

$$\frac{20kg\,(3m/s)^2}{10m} = F_N - 20kg\,(9.8m/s^2)$$

$$18N = F_N - 196N$$

$F_N = 214N$ = scale reading

At Top $\sum F = -\frac{mv^2}{r} = F_N - w$

$$-18N = F_N - 196N$$

$F_N = 178N$ = scale reading

Newton's Universal Law of Gravitation: $G\frac{m_1 m_2}{r^2}$

$F_{gravity} = G\left(\frac{M_1 m_2}{r^2}\right)$, where **G** is the gravitational constant (an empirically measured quantity), $\mathbf{m_1}$ and $\mathbf{m_2}$ are two different masses, and **r** is the distance between their centers of mass.

$$G = 6.67x10^{-11}\frac{Nm^2}{kg^2}$$

4. Calculate the force of gravity between a 100kg student and a 60kg student whose centers of mass are 1.7m apart.

$$F_{grav} = G\left(\frac{M_1 m_2}{r^2}\right) = 6.67\times10^{-11}\frac{Nm^2}{kg^2}\left(\frac{(100kg)(60kg)}{(1.7m)^2}\right)$$

$$F = 1.38\times10^{-7} N$$

5. Derive a formula for g, in terms of the earth's radius and mass.

Let M be Earth's mass and m = object mass

$$F_{gravity} = G\frac{M_1 m_2}{r^2}$$

$$F_{gravity} = W = mg$$

$$\cancel{m}g = G\frac{M\cancel{m}}{r^2}$$

$$g = G\frac{M}{r^2} \leftarrow \text{Earth mass}$$

6. Find the acceleration of gravity, g, using the Earth's mass ($5.972x10^{24}$kg) and average radius ($6.\cancel{731}x10^6$m).

6.371

$$g = G\frac{M}{r^2} = 6.67\times10^{-11}\frac{Nm^2}{kg^2}\left(\frac{5.972\times10^{24}kg}{(6.371\times10^6)^2}\right)$$

$$g = 9.8\ m/s^2$$

7. Derive a formula for the velocity of an object orbiting the Earth in a stable, circular orbit.

$$\Sigma F = \frac{mv^2}{r} = \frac{G M m}{r^2}$$

$$\Downarrow$$

$$v^2 = \frac{GM}{r} \Rightarrow v = \sqrt{\frac{GM}{r}} \leftarrow \text{Earth}$$

8. What is the velocity of a space station that is orbiting the Earth with an orbital radius of 30,000km?

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67\times10^{-11}\frac{Nm^2}{kg^2}(5.972\times10^{24}kg)}{3\times10^7 m}}$$

$$v = \sqrt{1.3\times10^7 m/s} = 3,640\ m/s$$

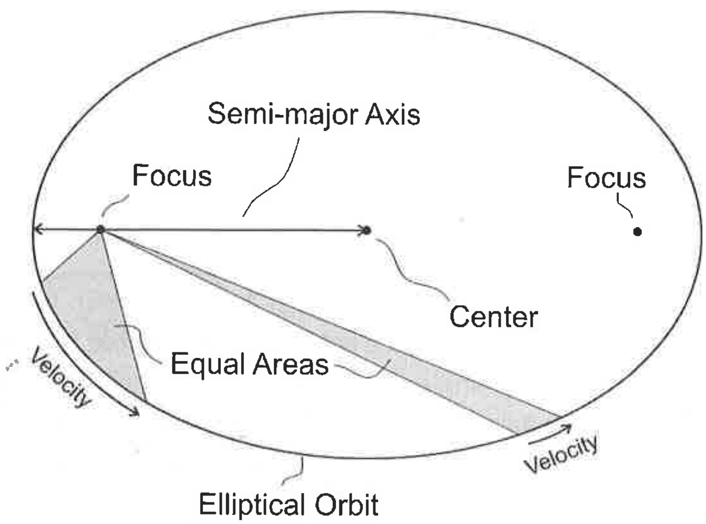
Kepler's Laws:

Necessary Conditions: a) Orbiting mass is much smaller than the orbited mass *(so the orbited mass is essentially stationary)*; b) The system is isolated from other masses.

1st Law -- Law of Orbits: All planets move in elliptical orbits, with the sun at one focus.

2nd Law -- Law of Areas: A line that connects a planet to the sun sweeps out equal areas in equal times.

3rd Law -- Law of Periods: The ratio of the squares of the periods of any two planets about the Sun is equal to the ratio of the cubes of their average distances from the Sun. For planets A and B with periods T_A and T_B and average distances r_A and r_B, orbiting around the same large mass, $\frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3}$.



9. Given that the Moon orbits Earth each 27.3 days and that it is an average distance of 3.84×10^8m from the center of Earth. Use Kepler's 3rd Law to calculate the period of an artificial satellite orbiting at an average altitude of 1500 km above Earth's surface.

Earth rad = 6,370 Km

$r_{satellite} = 7.87\times10^6 m$

A = Moon
B = Satellite

$$\frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3} \qquad \frac{(27.3\,days)^2}{T_B^2} = \frac{(3.84\times10^8 m)^3}{(7.87\times10^6 m)^3} \qquad \frac{745\,d^2}{T_B^2} = \frac{56.6\times10^{24}}{487\times10^{18}}$$

$$\frac{745\,d^2}{T_B^2} = 1.15\times10^5 \qquad T_B^2 = 6.48\times10^{-3}$$

$T_B = 0.080$ days

1 Astronomical Unit (AU) = Average distance from the center of the Earth to the center of the sun.

10. Neptune's period of revolution is 165 Earth years. What is its distance to the sun, in AU?

$$\frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3}$$

A = Earth
B = Neptune

$$\frac{(1yr)^2}{(165yr)^2} = \frac{(1Au)^3}{(r_B)^3} \qquad \frac{1}{27,200} = \frac{1Au^3}{(r_B)^3}$$

$$r_B^3 = 27,200\,Au^3$$

$$r_B = 30\,Au$$

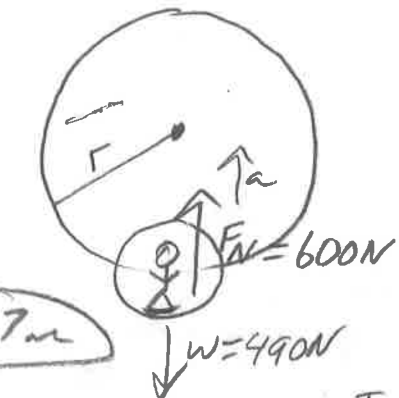
Practice Problems:

$V=$

Circular motion:

10. [Vertical Circles] A 50kg adult is standing on a bathroom scale inside a Ferris Wheel that is rotating at a constant rate of [redacted]. If the scale reads 600N when the adult is at the bottom of the Ferris wheel's circle, what is the wheel's radius?

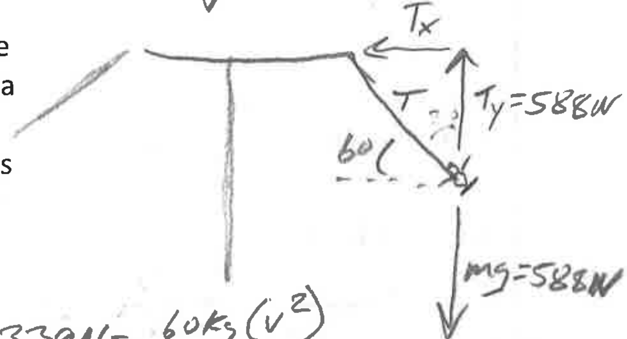
$4 m/s$



$$\Sigma F = \frac{mv^2}{r} = F_N - W = \frac{50kg(4m/s)^2}{r} = 110N$$

$$r = 7.27m$$

11. [Horizontal Circles in 2 Dimensions] A child in her seat are tethered to a rotating carnival ride by a cable that makes a 60° angle with horizontal. The child is 8m from the ride's axis of rotation. If the total mass of the seat + child equals 60kg and the cable has negligible mass, what is the speed of the child?



$$\Sigma F_x = \frac{mv^2}{r} = T_x \qquad T = \frac{588}{\sin 60°} = 679N$$

$$\Sigma F_y = 0 = T_y - mg \qquad T_x = T\cos 60° = 339N$$

$$339N = \frac{60kg(v^2)}{8m}$$

$$v = 6.73 m/s$$

Gravity:

12. Calculate the acceleration due to gravity on the surface of the sun, given the sun's mass (1.99×10^{30}kg) and radius (6.96×10^8m).

$$g = G\frac{M}{r^2} = 6.67\times10^{-11}\frac{Nm^2}{kg^2}\left(\frac{1.99\times10^{30}kg}{(6.96\times10^8 m)^2}\right) = 274 m/s^2$$

13. A satellite orbits the Earth in a stable orbit at a constant speed of 7,800m/s. What is the satellite's distance from the center of the Earth?

$$v^2 = \frac{GM_{Earth}}{r} \qquad (7{,}800 m/s)^2 = \frac{6.67\times10^{-11}\frac{Nm^2}{kg^2}(5.97\times10^{24}kg)}{r}$$

$$r = 6.55\times10^6 m$$

Kepler's 3rd Law:

14. Jupiter's distance from the sun is 5.2 AU (5x farther than the Earth-Sun Distance). How long is a year on Jupiter, in Earth years?

A B

$$\frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3} \qquad \frac{T_A^2}{(1yr)^2} = \frac{(5.2AU)^3}{(1AU)^3} \qquad \frac{T_A^2}{1yr^2} = \frac{141}{1}$$

$$T_A = 11.9 \text{ Earth yrs.}$$

15. Europa orbits the planet Jupiter once every 85.2 hours, at an average radius of 6.7×10^8m. Ganymede orbits Jupiter once every 172 hours. What is Ganymede's average orbital radius?

A B

$$\frac{(85.2h)^2}{(172h)^2} = \frac{(6.7\times10^8 m)^3}{r_B^3} \qquad 0.245 = \frac{3.00\times10^{26} m^3}{r_B^3}$$

$$r_B = 1.07\times10^9 m$$