

Physics 200 (Stapleton) Unit 2 Handouts

Name: _____

Notes - 3.1. Kinematics in Two Dimensions: An Introduction

1. Give three examples of 2-dimensional motion.

- A. _____
 B. _____
 C. _____

2. Given a right triangle of sides a and b and a hypotenuse of c , write the equation to find the length of c .

3. What is used to represent the magnitude and direction of a vector?

4. The length of the vector is directly proportional to the _____ of the vector.

5. HUGE IDEA: The horizontal and vertical components of two-dimensional motion are _____ of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

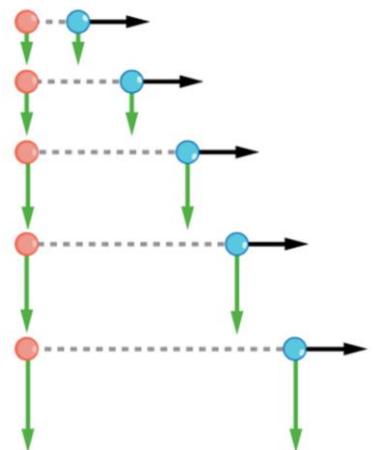
6. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. Which baseball hits the ground first?

7. For the thrown (blue) ball in Figure 3.6 (on the right),

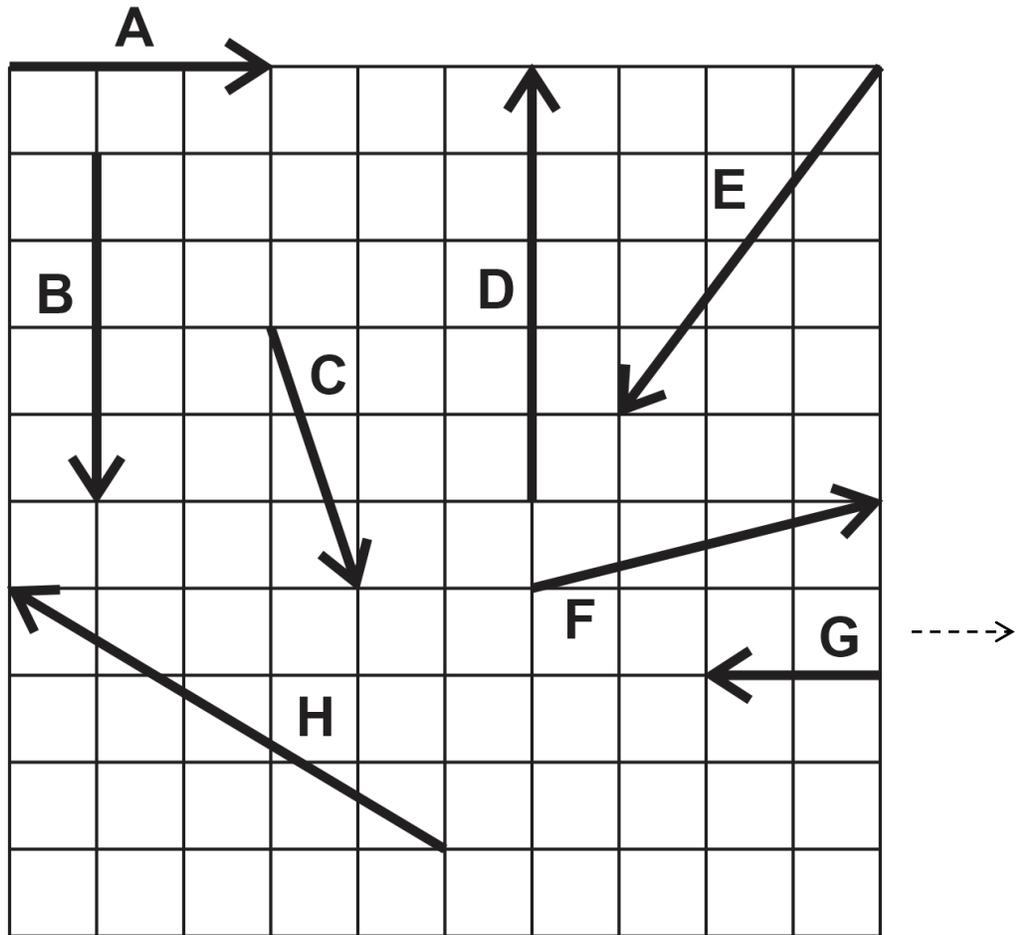
- A. Is there acceleration in the y -direction?
 B. Is there acceleration in the x -direction?

8. The key to analyzing such motion, called projectile motion, is to resolve (break) it into motions along perpendicular directions. Resolving two-dimensional motion into X and Y components is important, from a mathematical standpoint, because it allows us

to _____



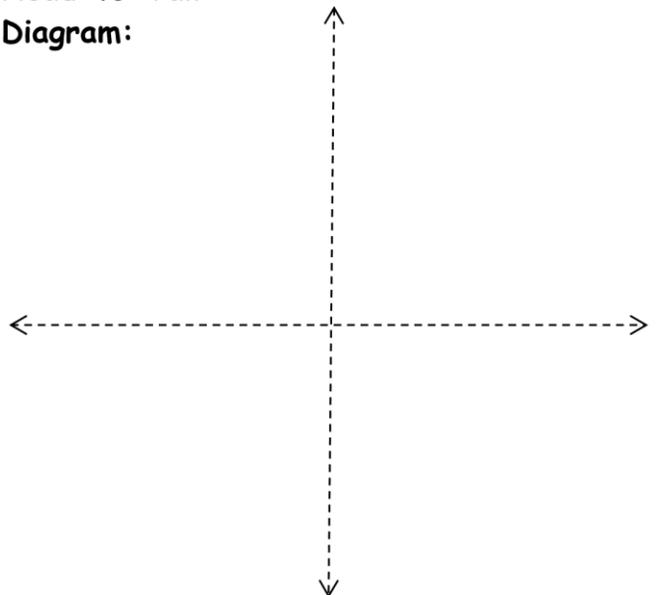
Vector Addition Practice:



1. Find the resultant vector that is produced by adding vectors A and B.

Vector	X comp.	Y comp.
Totals		
Magnitude of Resultant		
Direction of Resultant		

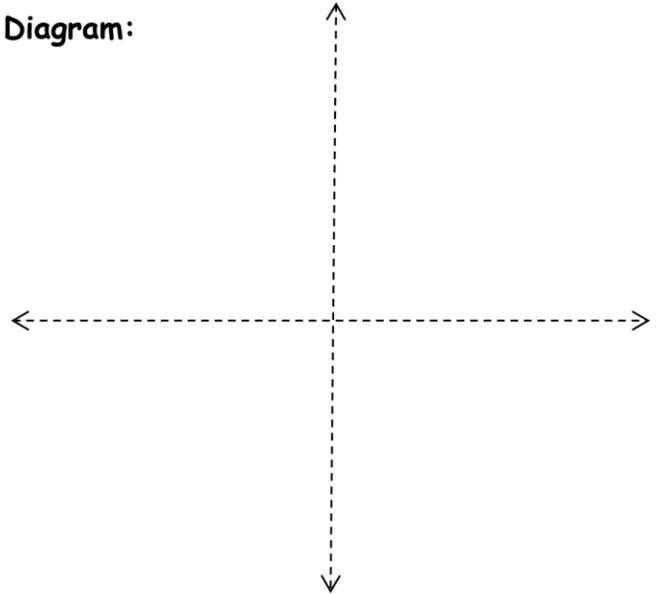
Head-to-Tail Diagram:



2. Add vectors E and C.

Vector	X comp.	Y comp.
Totals		
Magnitude of Resultant		
Direction of Resultant		

Head-to-Tail
Diagram:



3. What is the magnitude and direction of the resultant vector that is produced by adding vectors D, C, and A?

Find the resultant vectors from the additions of...

4. E + H
5. C + F
6. E + H + G

Answers:

1. 5, 53.1° below positive x axis	2. 7.3, 74° below negative x axis
3. 4.5, 26.5° above +x axis	4. 8.1, 7.1° below -x
5. 5.4, 21.8° below +x	6. 10.05, 84.2° left of -y axis

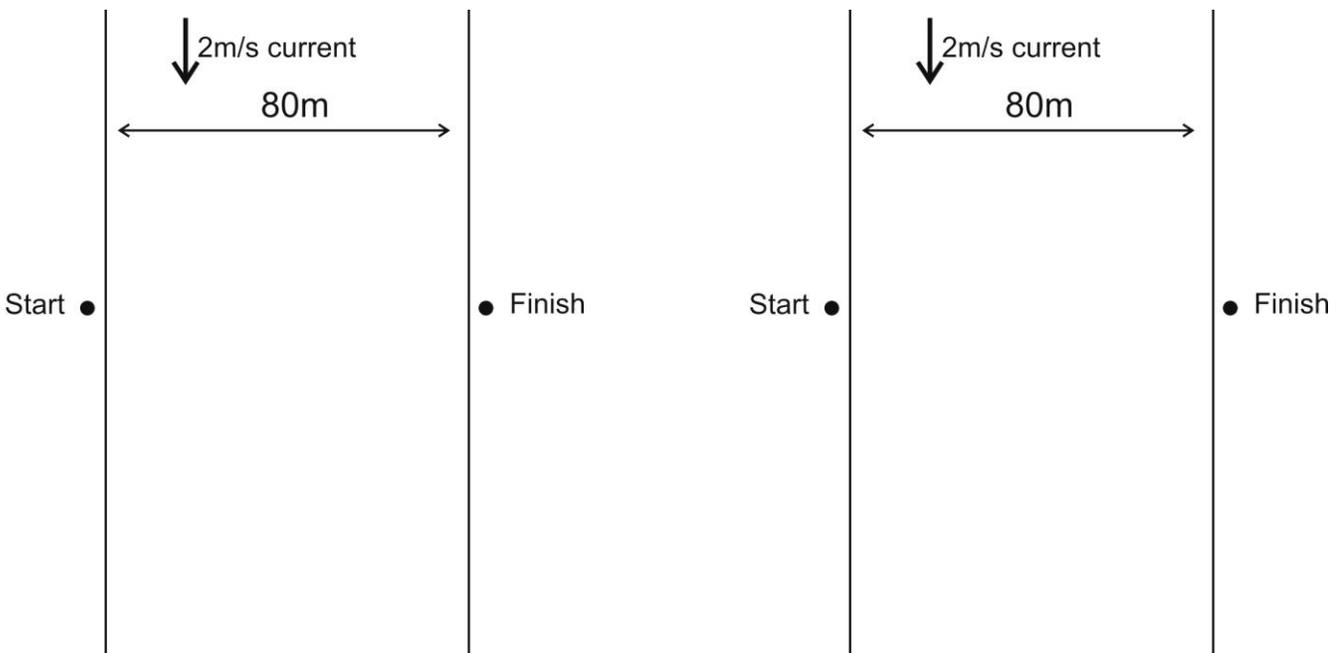
The Classic River Problem

Bob and Jane went camping. They put their canoe into a 80.0 meter wide river directly across from the perfect campsite. Avid hikers and paddlers, they are able to walk with a speed of 4.0 m/s and paddle their canoe with a speed of 3.0 m/s. The river flows South with a speed of 2.0 m/s.

Jane suggests that they angle the nose of their canoe into the current so that they travel in a line perpendicular to the current and make a landing at the campsite. She also offers to calculate (she never leaves home without her calculator) the correct angle that the compass should read as they paddle.

Bob declares that the fastest way to get across the river is to point the canoe directly across the river. Of course they will get washed downstream, but no matter says Bob, they can simply walk back to the campsite along the river.

1. Sketch both methods of travel. Label the component velocity vectors and the resultant velocity vectors with their correct magnitude. Proportions of vector magnitudes do not have to be perfect.
2. How long does Jane's method take?
3. How long does Bob's method take?



Still More River Problems

For each of the orthogonal problems below, create a head-to-tail diagram and use that diagram to answer the questions. For each non-orthogonal problem (a problem with at least one component vector that is not “vertical” or “horizontal”), create a table of x and y components, and use that table to answer the questions.

2. A motorboat is crossing a river that runs directly northward. The boat captain points the bow eastward and travels with a speed, relative to the water, of 4 m/s. The river is flowing at a rate of 7.0 m/s north. (from the Physics Classroom)
 - a. What is the resultant velocity of the motorboat?
 - b. If the width of the river is 100 meters wide, then how much time does it take the boat to travel shore to shore?
 - c. At what distance downstream does the boat reach the opposite shore?

3. A ferryboat (speed in still water = 5 m/s) needs to arrive at a point directly South across a 100 meter wide river. The river has a current of 0.50 m/s toward the east. Find the ferry’s time to cross. (from ChasePhysics)

4. A helicopter, flying where the average wind velocity is 38 km/h [25° N of E], needs to travel a displacement of 182 km [17° W of N] relative to the ground on a schedule of 2.0 h. Determine the required airspeed (speed through still air) and direction the helicopter must maintain. (from Northwestern University)

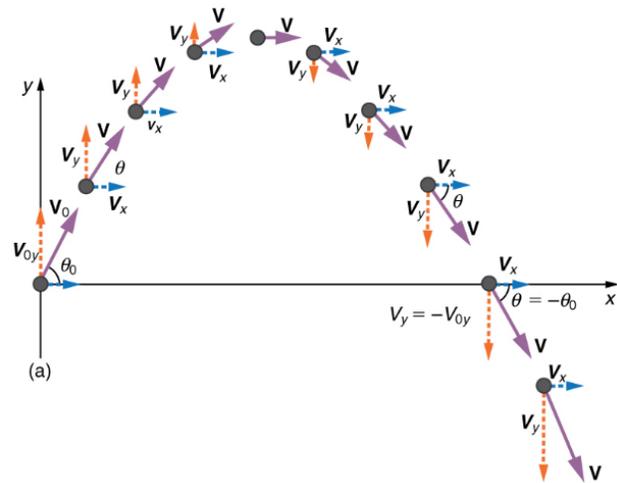
5. A boat travels westward at a rate of 3m/s. The boat is heading directly to the Northeast, and its speed in still water is 8m/s. What is the velocity of the water in which the paddler is paddling?

6. A quadcopter has a velocity of 20m/s in a direction 80 degrees west of North. The wind is blowing southward at a rate of 7 m/s. What are the quadcopter's airspeed and heading?

7. The driver of a golf cart on an aircraft carrier uses a compass to head northward. The cart's speedometer reads 6 mph. The aircraft carrier's heading is 28 degrees south of East, and its speed in still water is 8 mph. The ocean current is northwestward at a rate of 5mph. What is the actual velocity (relative to the Earth) of the golf cart?

Projectiles:

1. The velocity of a launched projectile can be resolved into vertical (y) and horizontal (x) components. What happens to each of these components during the flight of the projectile? Why? Assume that there is no air resistance.



2. A projectile is launched horizontally and to the left from the top of a tall building in the absence of air resistance. Sketch the path of the projectile as it falls to the ground. Use arrows to represent the object's speed, V_x , and V_y at the topmost point and at some other points during the fall.
3. Suppose a boat travels across a river, maintaining a heading that is due north. The boat's speed, relative to the water, is a steady 4m/s. The river's current has a westward velocity of 2m/s. What happens to the x and y components of the boat's velocity as it crosses the river? Sketch a diagram showing the boat's path. For at least three points, sketch vectors representing the boat's speed and velocity components (V_x and V_y).
4. What is the main difference between "river problems" and "projectile problems?"

5-9. A projectile is launched from ground level with an initial speed of V at an angle of θ above horizontal to the right. The projectile flies in the absence of air resistance until it returns to ground level. Remember Mr. Pennington's Huge Idea.

5. Create a sketch showing the initial conditions in this problem. Show the initial velocity vector. Also resolve the initial velocity vector into X and Y components and sketch those components.

6. Use trig identities to express the values of V_{ox} and V_{oy} .

$$V_{ox} =$$

$$V_{oy} =$$

7. Which component vector determines the time that the projectile remains in flight? Derive an equation for time aloft.

8. Derive an equation for the maximum height reached by the projectile.

9. Derive an equation for the x displacement of a projectile with the same starting and ending height. This is known as the **range formula**.

10. **Memorize this conversion:** $1\text{m/s} \approx 2.24\text{mph}$

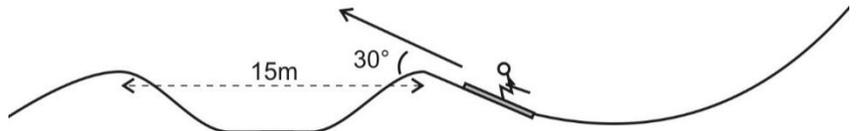
Projectile Practice Problems: Assume for all problems that there is no air resistance.

1. A car traveling at 60mph drives horizontally off of a cliff and falls to the ground 100m below.
 - a. Convert 60mph to m/s.
 - b. How long does it take the car to reach the ground?
 - c. How far, horizontally, does the car fly through the air?

2. You throw a ball at a 70° angle with an initial speed of 30mph. The ball flies in an arc and lands on a shelf at the same height from which you released it.
 - a. Convert 30mph to m/s.
 - b. Find the ball's v_{0x} and v_{0y} .
 - c. How long will the ball remain aloft before hitting the shelf?
 - d. What is the distance between the point of release and the point of impact on the shelf?
 - e. What maximum height was reached by the ball?

3. You are trying to throw a ball through an open window that is 20m above the point at which you will release the ball and 5m in front of that release point. To minimize possible damage, you want the ball to enter the window at its apogee (max height). At what angle and with what initial speed should you release the ball?

4. A skier builds a jump and a landing area as shown on the diagram to the right. The takeoff point and the landing point are 15m apart and at equal elevations. The jump is inclined to horizontal at a 30 degree angle.



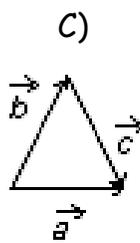
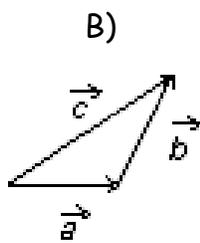
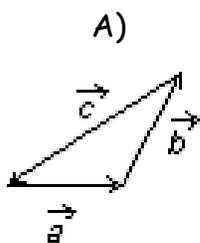
- What speed does the skier need to attain in order to travel exactly 15 meters?
- Given the initial speed from part a, what is the skier's maximum height, relative to the takeoff point?
- Given the same initial speed, what is the skier's time aloft?

Mr. Pennington's Chapter 3 Test 2015-2016

I. Multiple Choice: Select the one best answer for each question. Where g is used, assume it equals 10 m/s^2 and neglect air resistance for falling/moving objects.

- For a symmetric projectile with an initial velocity of v_0 , what other angle gives the same range as 60° ?
 A) 5° B) 30° C) 45° D) 60° E) 75°
- For a symmetric projectile with an initial velocity of v_0 , what angle gives the greatest range?
 A) 5° B) 30° C) 45° D) 60° E) 75°
- A projectile is shot vertically upward with a given initial velocity. It reaches a maximum height of 50.0 m . If, on a second shot, the initial velocity is tripled (i.e. $3X$), then the projectile will reach a maximum height of:
 A) 75 m B) 100 m C) 150 m D) 200 m E) 450 m

- The vectors \vec{A} , \vec{B} and \vec{C} are related by $\vec{C} = \vec{A} + \vec{B}$. Which diagram below illustrates this relationship?



- A bird flies at a speed of $15 \text{ meters per second}$ with respect to the ground and the wind is blowing at a speed of 5 m/s second with respect to the ground. [Note: The wind could be blowing with the bird, in the opposite direction of the bird or all other possible directions.] Which one of the speeds listed below is a possible net speed (i.e. vector sum) of the bird with respect to the ground?
 A) 3 m/s B) 5 m/s C) 9 m/s D) 18 m/s E) 25 m/s

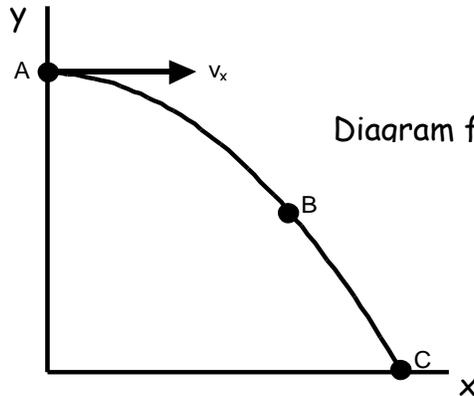
For #6-11, the answers will be $<$, $=$ or $>$, but you will mark A, B or C on your Scantron sheet. Assume no air friction for these projectiles and consider only the speed of the projectile (i.e. disregard the + and - signs).

A) $<$ B) $=$ C) $>$

6. v_B _____ v_{yA}

7. a_a _____ a_B

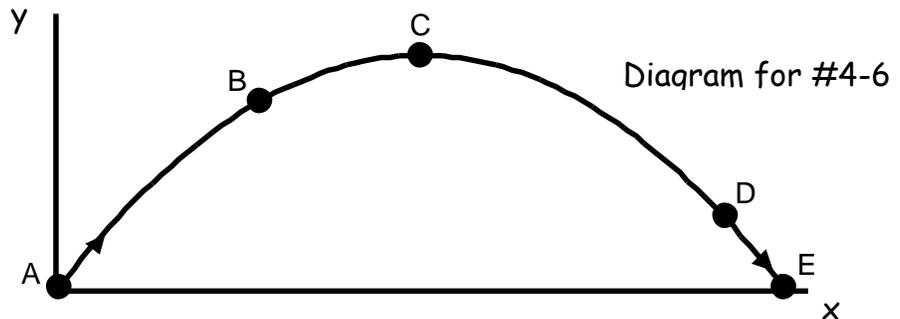
8. v_{yB} _____ v_{yC}



9. v_{xC} _____ v_{xD}

10. v_A _____ v_E

11. v_{yD} _____ v_{yB}



12. A vector has a component of 5 m in the +x direction and a component of 12 m in the +y direction. The magnitude of this vector is:

- A) 13 m B) 15 m C) 17 m D) 60 m E) 169 m

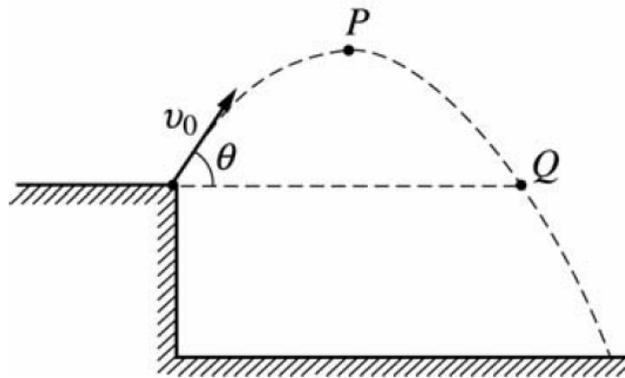
13. A vector in the xy plane has an x-component of 14.0 and a y-component of 9.4. The angle it makes with the positive x axis is:

- A) 26° B) 34° C) 45° D) 59° E) 66°

14. Which of the following cannot be a vector quantity?

- A) velocity B) acceleration C) force D) temperature

Diagram for
Questions 15 & 16



A rock is thrown from the edge of a cliff with an initial velocity v_0 at an angle θ with the horizontal as shown above. Point P is the highest point in the rock's trajectory and point Q is level with the starting point. Assume air resistance is negligible.

15. Which of the following correctly describes the horizontal and vertical speeds and the acceleration of the point at Point P?

	<u>Horizontal Speed</u>	<u>Vertical Speed</u>	<u>Acceleration</u>
A)	$v_0 \cos \theta$	0	g
B)	0	0	g
C)	$v_0 \cos \theta$	$v_0 \sin \theta$	g
D)	0	$v_0 \cos \theta$	g
E)	$v_0 \cos \theta$	0	0

16. Which of the following correctly describes the horizontal and vertical speeds and the acceleration of the point at Point Q?

	<u>Horizontal Speed</u>	<u>Vertical Speed</u>	<u>Acceleration</u>
A)	$v_0 \cos \theta$	0	g
B)	0	0	g
C)	$v_0 \cos \theta$	$v_0 \sin \theta$	g
D)	0	$v_0 \cos \theta$	g
E)	$v_0 \cos \theta$	0	0

17. A bullet shot horizontally from a gun. At the same instant, another bullet is simply dropped from the same height. Neglecting air resistance, the bullet shot from the gun

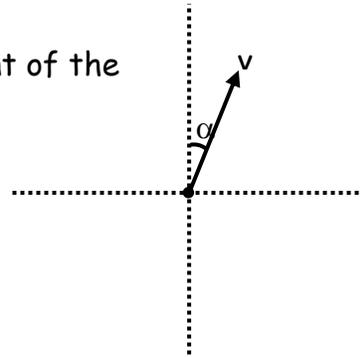
- A) strikes the ground much later than the dropped bullet.
- B) never strikes the ground.
- C) strikes the ground at the same time as the dropped bullet
- D) travels in a straight line.
- E) strikes the ground much sooner than the dropped bullet.

18. If θ is the angle with respect to the $+x$ -axis, the y -component of the vector A is given by

- A) $A \cos \theta$
- B) $\mu A \cos \theta$
- C) $A \sin \theta$
- D) $mg - A \sin \theta$
- E) $\tan^{-1}\theta$

19. Given the diagram to the right, what is the x -component of the vector v ?

- A) $V \sin \alpha$
- B) $V \cos \alpha$
- C) $V \tan \alpha$
- D) $V \sin^{-1}\alpha$
- E) $\sqrt{v_x^2 + v_y^2}$



II. Problems: Clearly show your work. Be neat. Use the correct number of significant figures and circle your answers. All answers must have units. Assume $g = 9.8 \text{ m/s}^2$.

1. A force vector \vec{F} has a magnitude of 12.0 N and a direction of 290.0° with respect to the +x axis.
 - A. Find the x-component of the force vector.
 - B. Find the y-component of the force vector.
2. A position vector \vec{r} has x- and y-components of $r_x = -6.50 \text{ m}$ and $r_y = 10.25 \text{ m}$.
 - A. Find the magnitude of the position vector.
 - B. Find the direction with respect to the +x axis of the position vector.
3. A projectile is launched at ground level with an initial speed of 40.0 m/s at an angle of 58.0° above the horizontal. It strikes a target above the ground 4.60 seconds later. [Note: This is not a symmetric projectile.] Assume the projectile is launched at $x_0 = 0.00 \text{ m}$ and $y_0 = 0.00 \text{ m}$.
 - A. What is the x-position of the target?
 - B. What is the y-position of the target?
4. A ball is kicked with an initial velocity of 16.0 m/s in the horizontal direction and 12.0 m/s in the vertical direction. [Note: This is a symmetric projectile.]
 - A. What maximum height is attained by the ball?
 - B. For how long does the ball remain in the air?
 - C. What is the range of the ball (i.e. the horizontal distance traveled)?
 - D. At what speed does the ball hit the ground?
5. A ball is thrown horizontally from the top of a 60.0-m building and lands 150.0 m from the base of the building. Ignore air resistance.
 - A. How long is the ball in the air?
 - B. What must have been the initial velocity?
6. A boat with a calm water speed of 10.0 m/s crosses a river $3.00 \times 10^2 \text{ m}$ wide. The boat keeps its bow pointed directly across the river, but drifts downstream because the river current is 3.00 m/s.
 - A. Find the time needed to cross the river.
 - B. Find the distance the boat drifts downstream.

Projectile Launcher Spreadsheet Practice Problems: For each problem, provide a satisfactory launch angle, muzzle velocity, and launcher setting. Then use the graph below to determine the necessary launcher setting. There are a variety of possible solutions to each problem. The easiest method is “guess and check.” You are expected to use the guess and check method, but feel free to rebel if you want to make your life more difficult.

1. The target and release point are at equal elevations. The horizontal distance from launcher to target is 5m. There is a 1m tall wall positioned at the midpoint (the 2.5m mark) between the launch pad and the target.

$$\Theta = \underline{\hspace{2cm}} \quad v_0 = \underline{\hspace{2cm}} \quad \text{Launcher Setting} = \underline{\hspace{2cm}}$$

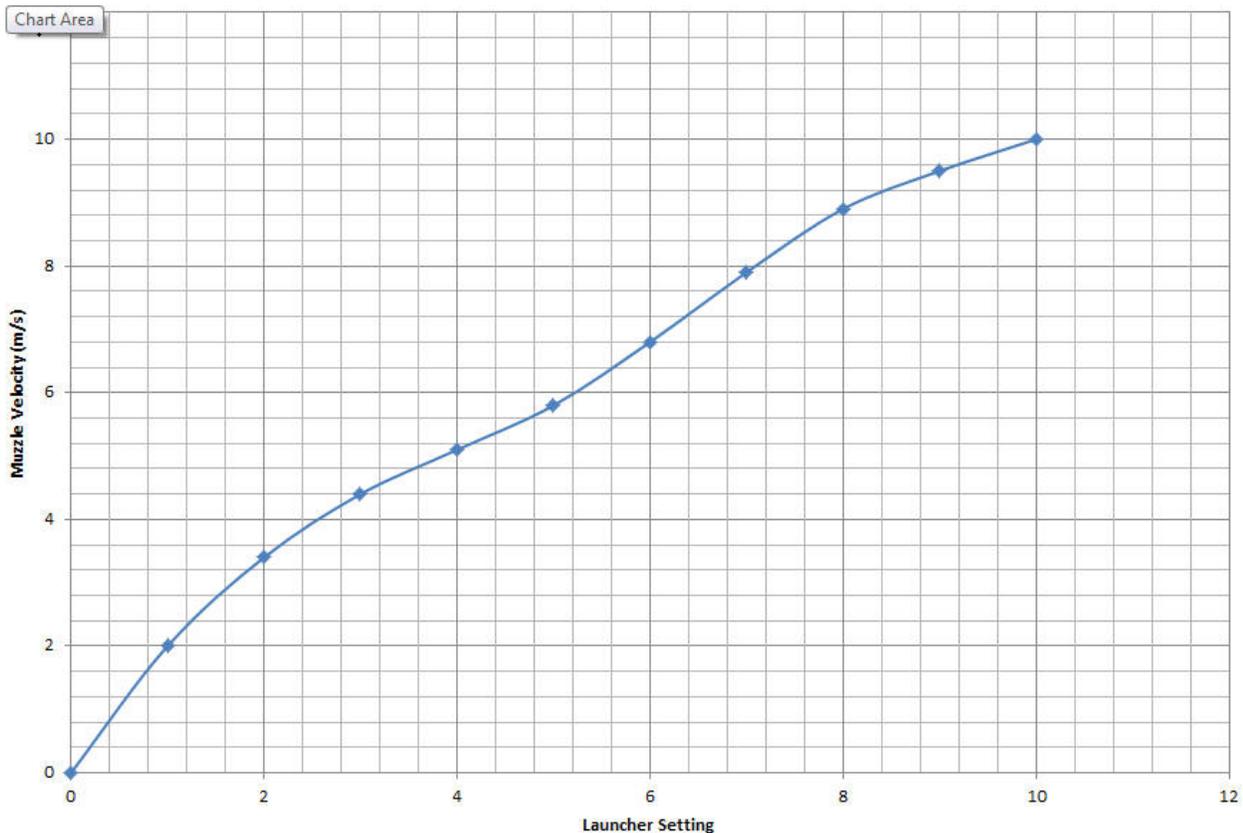
2. Horizontal distance to target = 3.5m. Target is 1.2m above release point.

$$\Theta = \underline{\hspace{2cm}} \quad v_0 = \underline{\hspace{2cm}} \quad \text{Launcher Setting} = \underline{\hspace{2cm}}$$

3. The target and release point are at equal elevations. The horizontal distance to the target is 4m. Three meters from the launcher (1m from the target), there is a wall with a window through which the projectile must be shot. The window is 0.5m tall, spanning a height between 0.75m and 1.25m from the ground.

$$\Theta = \underline{\hspace{2cm}} \quad v_0 = \underline{\hspace{2cm}} \quad \text{Launcher Setting} = \underline{\hspace{2cm}}$$

Launcher Calibration Graph



Chapter 3 Practice Test #2 – Kinematics in 2D

Concepts

1. Suppose a projectile is launched at some non-vertical angle in the absence of air resistance. The projectile remains in freefall for several seconds before hitting the ground.
 - a. Choose any non-vertical launch angle and sketch the flight path of the projectile.
 - b. Describe what happens to the projectile's V_x over time and explain why.
 - c. Describe what happens to the projectile's V_y over time and explain why.

2. Can a projectile's Y velocity affect the projectile's X displacement? Why or why not?

3. The velocity of an object moving in 2 dimensions has been resolved into two velocity components: $V_x=5\text{m/s}$ and $V_y = -5\text{m/s}$.
 - a. Describe the object's direction of travel.
 - b. Suppose the addition of a -5m/s air current in the y dimension doubles V_y to $V_y = -10\text{m/s}$. How does that change the object's **speed**?
 - c. How does this this air current (from part b) affect the object's X velocity?

4. Which of the following determine(s) the amount of time it takes for a boat to cross a river that runs in the positive X direction?
 - a. The river's current
 - b. The boat's Y velocity Component
 - c. The boat's X Velocity Component
 - d. All of these
 - e. None of these

5. The figure on the right shows three vectors. Two of the vectors are component vectors that add up to the resultant vector. Which vector is the resultant vector? How can you tell?

