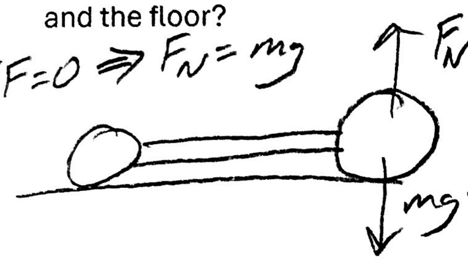
Individual Practice (Homework):

Front Wheel Radius (m)	0.025
Rear Wheel Radius (m)	0.08
Front Wheel and Axle Moment of Inertia (kgm^2)	$5x10^{-6}$
Drive Wheel and Axle Moment of Inertia (kgm^2)	$1.2x10^{-4}$
Total Car Mass (kg)	0.365
Mass of Both Rear Wheels (kg)	0.07
Mass of Both Front Wheels (kg)	0.015
Coefficient of Static Friction – Wheels on Floor	0.6
Motor Force at release point (N)	0.5
Rubber Band Stretch Distance – fishing line length (m)	0.25

The data table on the right relates to a different car. This car is a little longer, and its drive wheels are larger. Since it is longer, its rubber bands can stretch farther. Again, the designers have made sure that their motor delivers the most force possible without causing the drive wheels to spin out.

1. The rubber band car is symmetric from front to back, except for its wheels. The drive wheels have a total mass of 0.07kg, and the front wheels have a total mass of 0.015kg. The rest of the car has a mass of 0.28kg. What is the normal force exerted between the drive wheels and the floor?

$\Sigma F = 0 \Rightarrow F_N = mg$



$Weight = (0.07kg + 0.14 kg)(9.8 m/s^2) = 2.06 N$

Drive wheels

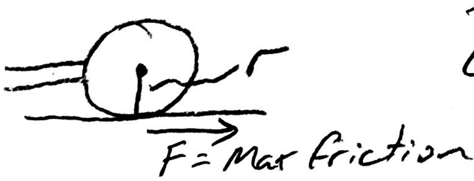
Half of "other mass")

$mg = F_N$

2. If the coefficient of static friction (μ_k) between the wheels and the floor is 0.6, what is the maximum force of friction that the wheels can exert against the floor (to make the car accelerate)?

$F_{Fr} \le \mu_s F_N \Rightarrow Max\, F_{Fr} = 0.6(2.06N) = 1.23\,N$

3. The car's drive wheels have a radius of 0.05m. What maximum torque can the drive wheels and axle exert without slipping and causing the car to "spin out?"



$\tau_{max} = 0.08m\,(1.23\,N) = 0.0988\,Nm$

Wheel Radius

Max Friction Force

4. The radius of the drive axle is 0.004m, and the fishing line winds around the axle. What is the maximum force that the fishing line can exert on the axle without causing the drive wheels to spin out? *Wheel and axle torque is the same at any radius

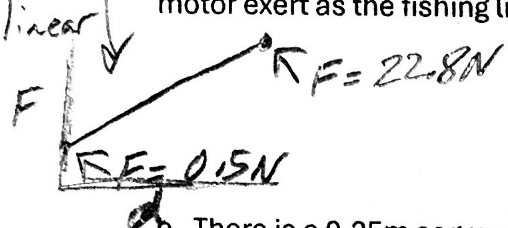
$\tau_{Max} = r_{Wheel} F_{Road} = r_{Axle} F_{Fishing\ line}$

Max Band Force

$\tau_{max} = 0.0988\,Nm = 0.004m(F_{line}) \Rightarrow F_{line} = 24.7N$

5. The students who made this car experimented with combinations of rubber band strands until they created a segment of band that can stretch all of the way to the drive axle before reaching the maximum force that you calculated in #4, above. When they first stretch their string to insert the release pin into the drive axle, the tension in the rubber bands is 0.5N.

a. Assuming that the rubber band performs like an ideal spring, what average force does the motor exert as the fishing line is wound up around the axle?



$$\text{Ave Force} = \frac{F_0 + F}{2} = \frac{0.5N + 24.7N}{2}$$

$$= 12.6N$$

b. There is a 0.25m segment of fishing line between the band and the drive axle, so winding up the car stretches the bands 0.25m. How much work is done on the bands during winding? *[This is the car's energy input, but the output is lower, because rubber bands are not 100% efficient.]*

$$W = Fd \Rightarrow W = \underbrace{12.6N}_{\text{Ave. Band Force}}\underbrace{(0.25m)}_{\text{Stretch Distance}} = \underbrace{3.15J}_{\text{Input E}}$$

c. Let's estimate that this car is 60% efficient. How much kinetic energy will it have when it reaches its top speed?

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \Rightarrow 0.6 = \frac{\text{Output}}{3.15J} \Rightarrow \text{output} = 1.89J$$

6. Write an equation that we can use to solve for the car's top speed (v). Set the kinetic energy from part C of the previous question equal to the sum of the kinetic energies of the car when it is at its top speed. Substitute linear quantities for angular quantities so that we can solve for V. Then find the data that you need in the table and solve for maximum velocity.

*D="Drive"
F="Front"

$$KE_{Tot} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I_D\omega_D^2 + \tfrac{1}{2}I_F\omega_F^2$$

$$KE_{Tot} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I_D\frac{v^2}{r_D^2} + \tfrac{1}{2}I_F\frac{v^2}{r_F^2}$$

$$2KE_{Tot} = v^2\left(m + \frac{I_D}{r_D^2} + \frac{I_F}{r_F^2}\right)$$

Units Left out to Save Space

$$V_{max} = \sqrt{\frac{2KE}{m + \frac{I_D}{r_D^2} + \frac{I_F}{r_F^2}}} = \sqrt{\frac{2(1.89)}{0.365 + \frac{1.2\times10^{-4}}{(0.08)^2} + \frac{5\times10^{-6}}{(0.025)^2}}}$$

$$V_{max} = \sqrt{\frac{3.78}{0.365 + 0.0188 + 0.008}} = \boxed{3.11\ m/s}$$