

## Unit 6 Handout #2: Rotational Motion

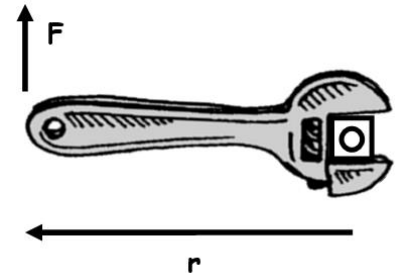
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### Torque Notes

#### I. Torque

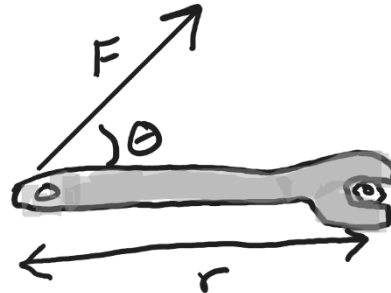
A. The rotational equivalent of force is \_\_\_\_\_. Its symbol is \_\_\_\_\_.

B. **Torque** =  $rF$  = lever arm ( $r$ ) x perpendicular force ( $F$ ).



C. When  $F$  is not perpendicular to  $r$ ,

Torque = \_\_\_\_\_

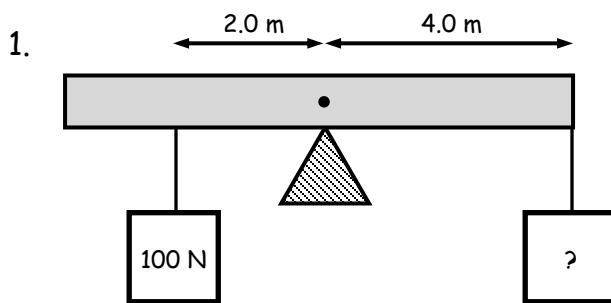


#### II. Rotational Equilibrium

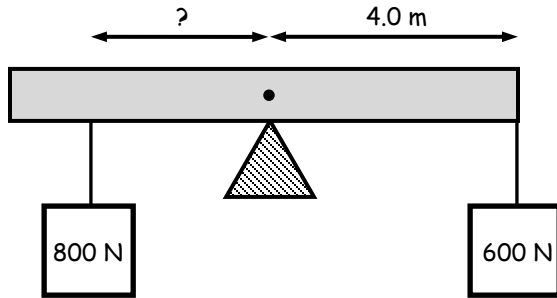
A. In rotational equilibrium,  $\sum \tau_i =$

In other words, the clockwise torques = the counterclockwise torques

B. Examples of rotational equilibrium:



2.



### Notes -Angular Speed and Acceleration

1. What is the definition of angular speed  $\omega$ ? What are the units of  $\omega$ ?
2. How are linear velocity and angular speed related?
3. What is the definition of angular acceleration  $\alpha$ ? What are the units of  $\alpha$ ?
4. Use a diagram to distinguish between tangential acceleration ( $a_t$ ) and centripetal acceleration ( $a_c$ )?
5. How are tangential acceleration and angular acceleration related?
6. How are linear acceleration, tangential acceleration, and angular acceleration related?

## Notes - 10.3 Dynamics of Rotational Motion: Rotational Inertia

1. Rotational Inertia (a.k.a. moment of inertia) is a rotational version of \_\_\_\_\_. Whereas mass and ordinary inertia cause resistance to linear acceleration, an object's moment of inertia describes its resistance to \_\_\_\_\_. The rotational inertia of an object depends both on its mass and the distance of that mass from the object's axis of rotation. As an example, consider a door. If the door's mass is increased, it will have a \_\_\_\_\_ (higher, lower) resistance to rotational acceleration, and its moment of inertia will be \_\_\_\_\_ (higher, lower). If the door's mass is shifted "inward," so that it is closer to its axis of rotation, the door will have a \_\_\_\_\_ (higher, lower) resistance to rotational acceleration, and its moment of inertia will be \_\_\_\_\_ (higher, lower). The opposite is true if the door's mass is shifted away from its hinge.

2. Starting with Newton's 2<sup>nd</sup> Law, derive an expression for torque  $\tau$  in terms of mass  $m$ , lever arm  $r$  and angular acceleration  $\alpha$  (and introduce  $I$  - "Rotational Inertia" or "moment of inertia")

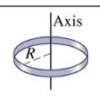
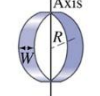
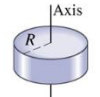
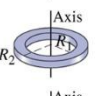
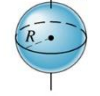
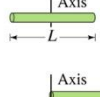
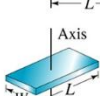


3. The two definitions of torque:

4. Rotational Inertia ( $I$ ) of Various Objects

A. A single point mass:

B. Multiple point masses:

C. Specific shapes - see chart or web search

Object	Location of axis		Moment of inertia
(a) Thin hoop, radius $R$	Through center		$MR^2$
(b) Thin hoop, radius $R$ width $W$	Through central diameter		$\frac{1}{2}MR^2 + \frac{1}{12}MW^2$
(c) Solid cylinder, radius $R$	Through center		$\frac{1}{2}MR^2$
(d) Hollow cylinder, inner radius $R_1$ outer radius $R_2$	Through center		$\frac{1}{2}M(R_1^2 + R_2^2)$
(e) Uniform sphere, radius $R$	Through center		$\frac{2}{5}MR^2$
(f) Long uniform rod, length $L$	Through center		$\frac{1}{12}ML^2$
(g) Long uniform rod, length $L$	Through end		$\frac{1}{3}ML^2$
(h) Rectangular thin plate, length $L$ , width $W$	Through center		$\frac{1}{12}M(L^2 + W^2)$

### Notes - 10.4 Rotational Kinetic Energy

1. Starting with the linear (or tangential) kinetic energy formula, derive a formula for the rotational kinetic energy of a single mass  $m$ , with a velocity  $v$ , revolving around an axis at a radius  $r$ . The formula should be in terms of  $I$  and  $\omega$ .
2. Calculate the final speed of a **solid cylinder** that rolls down a 2.00-m-high incline. The cylinder starts from rest, has a mass of 0.750 kg, and has a radius of 4.00 cm.
3. Calculate the final speed of a **hoop** of the same radius (4cm) that is allowed to roll down an incline of the same height (2m)

4. Find the moment of inertia of a rubber band car rear wheel and axle (2 wheels, plus an axle, connected together) that achieves a final velocity of 3m/s after rolling down a ramp from a height of 1m. The wheel and axle has a radius of 0.06m and a mass of 0.12kg.

### **Notes - 10.5 Angular Momentum and Its Conservation**

1. Write the equation for linear momentum.
2. Write the equation for angular momentum.
3. State the Law of Conservation of Angular Momentum in words.
4. Write the equation for the Conservation of Angular Momentum.

5. Suppose an ice skater is spinning at  $0.800 \text{ rev/s}$  with her arms extended. She has a moment of inertia of  $2.34 \text{ kg}\cdot\text{m}^2$  with her arms extended and a moment of inertia equal to  $0.363 \text{ kg}\cdot\text{m}^2$  with her arms close to her body. (These moments of inertia are based on reasonable assumptions about a  $60.0\text{-kg}$  skater.)

A. What is her initial angular velocity, in  $\text{rad/s}$ ?

B. What is her initial angular momentum?

C. What is her final angular velocity?

B. What is her rotational kinetic energy before and after she does this? Why does her  $KE_R$  change?

## **Notes:** Rubber Band Car Calculations with Energy

### **Notes Part 1: MOI Determination, Method 2:**

Some students use a different method to find the moment of inertia of their drive wheel and axle. They remove the wheel and axle from their car and let it roll down a ramp. They release it from rest and record its vertical drop, time to descend, mass, and outer radius.

Linear Distance Rolled (m)	2.8
Drive Wheel Radius(m)	0.08
Time to roll down ramp (s)	2.5
Drop in height during descent down ramp (m)	0.28
Wheel and Axle Mass (kg)	0.12

1. What is the average linear velocity of the wheel and axle as it rolls down the ramp?
2. What is the final linear velocity of the wheel and axle when it reaches the bottom of the ramp?
3. Use the law of conservation of energy to write an equation comparing the wheel and axle's energy at the top of the ramp to its energy at the bottom. Assume that there is no non-conservative work done.
4. Rewrite the equation, converting to all "linear" (not rotational) terms – except for moment of inertia (I).
5. Plug-in the givens (data above) and solve for the moment of inertia of the wheel and axle.



## **Practice: Rubber Band Car Calculations with Energy**

### **Practice Part 1: MOI Determination, Method 2:**

Some students use a different method to find the moment of inertia of their drive wheel and axle. They remove the wheel and axle from their car and let it roll down a ramp. They release it from rest and record its vertical drop, time to descend, mass, and outer radius.

Linear Distance Rolled (m)	1.21
Drive Wheel Radius(m)	0.051
Time to roll down ramp (s)	1.79
Drop in height during descent down ramp (m)	0.15
Wheel and Axle Mass (kg)	0.0878

1. What is the average linear velocity of the wheel and axle as it rolls down the ramp?
2. What is the final linear velocity of the wheel and axle when it reaches the bottom of the ramp?
3. Use the law of conservation of energy to write an equation comparing the wheel and axle's energy at the top of the ramp to its energy at the bottom. Assume that there is no non-conservative work done.
4. Rewrite the equation, converting to all "linear" (not rotational) terms – except for moment of inertia (I).
5. Plug-in the givens (data above) and solve for the moment of inertia of the wheel and axle.

