Unit 6: Momentum/Impulse/Collisions

Physics 200, 25-26 Name: Key

Notes - 8.1 Linear Momentum and Force

1. Write the symbol and equation for momentum.

$$p = \underset{\text{mass}}{\underset{\uparrow}{m}}\overset{\text{velocity}}{\overset{\swarrow}{v}}$$

2. Why is the symbol for momentum a lowercase p?

It derives from petere -- latin for "to push"

3. What are the units for momentum?

$$\underset{\text{mass}\nearrow}{\text{kg}}\;\underset{\nwarrow\text{velocity}}{\text{m/s}}$$

4. Calculate the momentum of a 110-kg football player running at 8.00 m/s.

$$p = mv = 110\text{kg}(8\text{m/s}) = \boxed{880\text{ kgm/s}}$$

Notes - 8.2 Impulse

5. Use Newton's 2nd Law and the basic acceleration formula to write an equation for Δp in terms of Force and time.

$$\left.\begin{matrix} F_{Net} = ma \\ a = \dfrac{\Delta v}{\Delta t} \end{matrix}\right\} \Rightarrow F_{Net} = \frac{m\Delta v}{\Delta t} \Rightarrow \boxed{F_{Net}\Delta t = \Delta p} \leftarrow \text{"classic" formula}$$

or

$$\Rightarrow \boxed{F_{Net}\Delta t = m\Delta v} \leftarrow \text{often more useful}$$

6. $F_{net}\,\Delta t$ (more commonly written as Ft) is called Impulse.

7. Impulse is equivalent to a change in momentum.

8. Imagine a ball falling to the floor and then bouncing upward to a height of 40cm. Now imagine someone throwing the same ball upward a height of 40cm. In which case is a greater impulse applied to the ball? Why?

Bounce demonstrates more impulse, because momentum changes more, due to reversal of velocity.

$$Ft = \boxed{m\Delta v} \leftarrow \text{changes more with bounce}$$

9. The effect of a force on an object depends on the force's magnitude and duration. A very large force acting for a short time will have a great effect on the momentum of a tennis ball. A small force could cause the same change in momentum, but it would have to act for a longer time.

10. Use the impulse formula to show how the same change in momentum can be accomplished by a variety of forces and times.

$$\Delta p = Ft = F_t = {}_F t$$

$$\nwarrow \quad \uparrow \quad \nearrow$$

Different ways to get the same Δp

11. **Example Problem:** Suppose a 60kg human is falling from the sky at a rate of 20m/s. If the human is brought to a stop by hitting the bare ground, the average force applied to the person during impact is 24,000N. What is the duration over which the impact force is applied?

$$\boxed{Ft = m\Delta v} \Rightarrow 24{,}000N(\Delta t) = 60kg(20m/s)$$

$$\boxed{\Delta t = 0.05s}$$

12. Describe a few ways in which impulse can be manipulated to used to save lives:

$\Delta p = {}^F t$ — padded helmets, air bags, water landings

"soft" collisions

$\Delta p = F_t$ — Breaking windows to escape burning building or sinking car

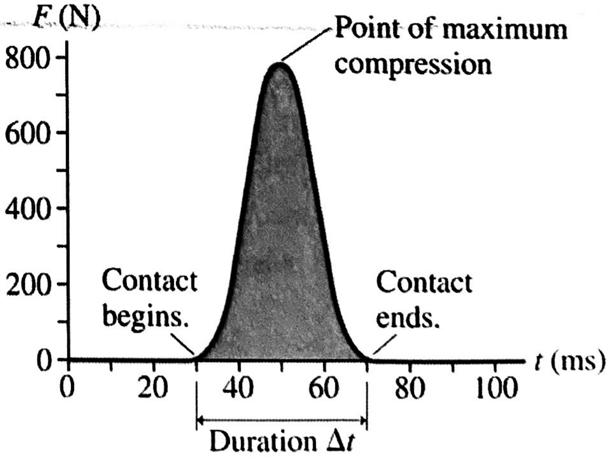
Self-defense

"hard" — collisions

13. What does the area under a force-time graph represent? Can you guess what was being graphed here?

Impulse (Ft)

Force exerted on a ball bouncing on a floor



Notes - 8.3 Conservation of Momentum

14. Since $F_{net}\,\Delta t = \Delta p$, whenever $F_{net} = 0$, Δp = 0 (i.e. the total momentum is constant).

15. **Law Of Conservation of Momentum:** For any isolated mechanical system, total momentum in the system is constant [An isolated mechanical system is one that does not experience any external forces (in other words, $F_{net} = 0$)].

16. Another way to look at this is to apply Newton's 3rd Law to prove that momentum is conserved during any collision. When two objects collide...

Ⓐ→←Ⓑ

- The objects experience equal and opposite forces, and
- They experience the forces for equal times (Δt),
- So they experience equal and opposite impulses,
- So they experience equal and opposite Δp,
- So their Δp cancel one another, and P_{net} is unchanged.

17. Conservation of momentum formula for 2 objects (This does not apply if there is an external force acting on the system. In that case, an outside force adds or removes momentum to or from the total.)

$$M_{1i}V_{1i} + M_{2i}V_{2}{}^{i} = M_1V_1 + M_2V_2$$

m_1 V_{1i} V_{2i} M_2

18. **Example Problem:** A 3kg object has a velocity of 2m/s before it crashes into a second object that has been traveling with a velocity of -5m/s. After the collision, the 3kg object has a velocity of 1m/s and the other object has a velocity of 2m/s. What is the mass of the second object? V_1 V_2

$$3kg(2m/s) + M_2(-5m/s) = 3kg(1m/s) + M_2(2m/s)$$

$$6kgm/s - 5m/s(M_2) = 3kgm/s + 2m/s(M_2)$$

$$3kgm/s = 7m/s(M_2)$$

$$M_2 = 0.43kg$$

Notes - 8.4 & 8.5 Elastic and Inelastic Collisions

19. Characteristics of **Elastic** (a.k.a. perfectly elastic) Collisions:

- Kinetic energy is conserved (100% Efficient)
- "Separation Speed" is equal to "Closing Speed"
- Thermal energy is not produced

Rate at which separation distance increases

rate at which separation distance decreases

20. Characteristics of **Inelastic** Collisions:

- Kinetic energy decreases
- "Separation Speed" is less than "Closing Speed"
- Thermal energy is produced

21. Give some examples of nearly elastic collisions between macroscopic objects

Newton's Cradle, Billiard Balls

22. Give an example of a totally inelastic (a.k.a. perfectly inelastic) collision:

- Objects stick together
- separation speed = 0

⇒ Arrow sticking in target, Baseball being caught in glove

Momentum and Impulse problems

1. What is the magnitude of the momentum of a 28-g sparrow flying with a speed of 8.4 m/s?

2. A constant friction force of 25 N acts on a 65-kg skier for 20s. What is the skier's change in velocity?

3. A 0.145-kg baseball pitched at 39.0 m/s is hit in a horizontal line drive straight back toward the pitcher at 52.0 m/s. If the contact time between bat and ball is 3.00×10^{-3}s, calculate the average force between the ball and bat during contact.

Problems

1. What is the magnitude of the momentum of a 28-g sparrow flying with a speed of 8.4 m/s?

$p = mv = (.028\,\text{kg})(8.4\,\frac{m}{s}) = 0.235\ \text{kg}\,m/s$

2. A constant friction force of 25 N acts on a 65-kg skier for 20 s. What is the skier's change in velocity?

$F\Delta t = m\Delta v$

$(25)(20) = 65\,\Delta v$

$\Delta v = 7.69\,m/s$

3. A 0.145-kg baseball pitched at 39.0 m/s is hit on a horizontal line drive straight back toward the pitcher at 52.0 m/s. If the contact time between bat and ball is $3.00\text{x}10^{-3}$s, calculate the average force between the ball and bat during contact.

$F\Delta t = m\Delta v$ $\quad\quad \Delta v = v' - v = 52 - {}^{-}39 = +91\,m/s$

$F(3\times10^{-3}) = (.145)(91)$

$F = 4398\,N$

4. Calculate the force exerted on a rocket, given that the propelling gases are expelled at a rate of 1500 kg/s with a speed of $4.00\text{x}10^{4}$m/s (at the moment of takeoff). The force on the gas can be found from its change in momentum.

$F\Delta t = m\Delta v$ $\quad\quad F = (1500\,\frac{kg}{s})(4\times10^{4}\,\frac{m}{s})$

$F = \frac{m\Delta v}{\Delta t}$ kg/s $\quad\quad F = 6.0\times10^{7}N$

5. A golf ball of mass 0.045 kg is hit off the tee at a speed of 45 m/s. The golf club was in contact with the ball for 3.5×10^{-3} s. Find (*a*) the impulse imparted to the golf ball, and (*b*) the average force exerted on the ball by the golf club.

$F\Delta t = m\Delta v$

$m\Delta v = (.045)(+45) = 2.025 =$ impulse and momentum

Kg m/s

$F\Delta t = 2.025$

$F = 579\,N$

6 You are the design engineer in charge of the crashworthiness of new automobile models. Cars are tested by smashing them into fixed, massive barriers at 50 km/h (30 mph). A new model of mass 1500 kg takes 0.15 s from the time of impact until it is brought to rest. (*a*) Calculate the average force exerted on the car by the barrier. (*b*) Calculate the average deceleration of the car.

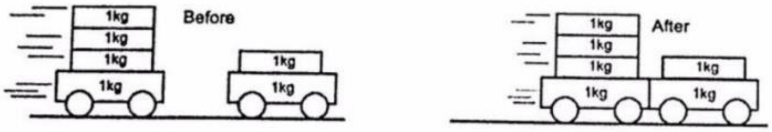
$\Delta V = 0 - 13.9 = -13.9 \frac{m}{s}$

$F\Delta t = m\Delta v$

$F(.15) = (1500)(-13.9)$

$F = 139{,}000 N$

$a = \frac{\Delta v}{\Delta t} = 93 m/s^2$



7. v=3m/s v=0m/s v=?

$(4kg)(3m/s) + 2kg(0) = 6kg(v) \Longrightarrow V = \frac{12kg\,m/s}{6kg} = \boxed{2m/s}$

8. v=0m/s v=-2m/s v=?

$(3kg)(0m/s) = (2kg)(-2m/s) + (1kg)(v) \Longrightarrow V = \boxed{4m/s}$

9. v=5m/s v=2m/s v=?

$(3kg)(5m/s) + (2kg)(2m/s) = 5kg(v) \Longrightarrow v = \boxed{3.8m/s}$

10 1. A child in a boat throws a 6.40 kg package out horizontally with a speed of 10.0 m/s. Calculate the velocity of the boat immediately after, assuming that it was initially at rest. The mass of the child is 26.0 kg, and that of the boat is 45.0 kg. Ignore water resistance.

The throwing of the package is a momentum-conserving action, if the water resistance is ignored. Let "A represent the boat and child together, and let "B" represent the package. Choose the direction that the package is thrown as the positive direction. Apply conservation of momentum, with the initial velocity of both objects being 0.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow (m_A + m_B)v = m_A v'_A + m_B v'_B \rightarrow$$

$$v'_A = -\frac{m_B v'_B}{m_A} = -\frac{(6.40\text{ kg})(10.0\text{ m/s})}{(26.0\text{ kg} + 45.0\text{ kg})} = \boxed{-0.901\text{ m/s}}$$

11 2. A 12,600-kg railroad car travels alone on a level frictionless track with a constant speed of 18.0m/s. A 5350-kg load, initially at rest, is dropped onto the car. What will be the car's new speed?

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_A v_A + m_B v_B = (m_A + m_B)v' \rightarrow$$

$$v' = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{(12,600\text{ kg})(18.0\text{ m/s}) + 0}{(12,600\text{ kg}) + (5350\text{ kg})} = \boxed{12.6\text{ m/s}}$$

13 12. Boat A has a mass of 10kg and a velocity of 3m/s. Boat B has a mass of 15kg and a velocity of -1m/s. The two boats collide and bounce away from one another. After the bounce, boat B has a velocity of 1.4m/s.

Conservation of Momentum

$(10\text{kg})(3\text{m/s}) + 15\text{kg}(-1\text{m/s}) = 10\text{kg}(V_A) + (15\text{kg})(1.4\text{m/s})$

$15\text{kgm/s} = 10\text{kg}(V_A) + 21\text{kgm/s}$

$V_A = \frac{-6\text{kgm/s}}{10\text{kg}} = -0.6\text{m/s}$

a. What is the velocity of boat A after the bounce?

-0.6m/s

b. What impulse is experienced by boat A during the collision?

$F\Delta t = \Delta p = P_{Final} - P_{initial} = (-0.6\text{m/s})(10\text{kg}) - (10\text{kg})(3\text{m/s}) = -36\text{kgm/s}$

c. What impact force is felt by boat A

$F(0.1\text{s}) = -36\text{kg m/s}$ $F = 360\text{N}$

d. What impulse is experienced by boat B?

36kgm/s

e. What impact force is felt by boat A?

$F = -360\text{N}$

Action/reaction

Impulses are = and opposite, since forces are = + opposite, and times are equal.

g. Is the collision elastic or inelastic?

separation speed ($1.4\text{m/s} - 0.6\text{m/s} = 2\text{m/s}$)
is less than
closing speed ($3\text{m/s} - (-1\text{m/s}) = 4\text{m/s}$),
so KE is lost. You can also show this by calculating $KE_0 + KE$.

Physics 200 (Stapleton) Name: Key
25-26 Practice Test: Momentum and Impulse

Multiple Choice

1. Which of the following are correct units for momentum?
[✓ a. Kgm/s] b. N/ms^2 c. Nm d. Nm/s e. Kgm/s^2

2. Which other units are also acceptable units for momentum?
a. Js b. kW/s [✓ c. Ns] d. JKg e. Kgm^2/s

Piñata → $Ft = m\Delta v$
$\Delta v = 1m/s - 2m/s = -1m/s$
$Ft = 2kg(-1m/s) = -2kgm/s$
Piñata Impulse -2Ns

3. A 2kg pinata is initially swinging with a velocity of 2m/s. A stick, moving with a velocity of -10m/s, hits the pinata, causing the pinata's velocity to change to 1m/s. During this collision, what impulse is imparted to the **stick**?
A. 1Ns [✓ B. 2Ns] C. 3Ns D. 4Ns E. 5Ns

Stick Impulse is equal and opposite

4. A motionless bean in a microwave suddenly explodes into two parts that fly in opposite directions. One part has a mass of 0.6g, and it flies away with a speed of 12m/s. The other part has a mass of 0.4g. If there are no outside forces acting on the system of the bean, what is the speed of the 0.4g bean fragment when it flies away?
[✓ A. 18m/s] B. 20m/s C. 22m/s D. 24m/s E. 26m/s

$\Sigma P_0 = 0kgm/s$ $P_1 = 0.6g(12m/s) = 7.2gm/s$ → so $P_2 = -7.2gm/s$
$-7.2gm/s = 0.4g(v)$
$v = -18m/s$

5-7. In the game of *egg toss*, someone tosses a raw egg to you from a great distance, and you try to catch it without breaking the egg. Your friend holds their hands still to catch the egg, stopping the egg immediately as it reaches their hands. You catch your egg by moving your hands backward as the egg makes contact, bringing the your egg to rest more slowly than your friend's egg. The two eggs have exactly the same mass (70g), and they arrive at your hand and your friend's hand with the same velocity (10m/s).

5. What happens to your egg's momentum as you catch it?
a. Its momentum increases [✓ b. Its momentum decreases] to zero c. no change

6. Whose egg experiences the greatest force during the catch?
[✓ a. Your Friend's Egg] b. Your egg
c. Neither; they're the same d. There's no way to tell for certain.

$\Delta p = Ft$ $\Delta p = Ft$

7. Whose egg experiences the greatest impulse during the catch?
a. Your Friend's Egg b. Your egg [✓ c. Neither; they're the same] d. Can't tell

Impulse = Δp same; they both decrease to zero.

8-10. The table on the right shows data collected from collisions of carts with a stationary (non-moving) force sensor. Each of these carts collided with the sensor at **exactly the same closing speed**, and each of these carts had exactly the same **mass (0.2kg). Exactly one** of these collisions was **totally inelastic.** Based on this information…

Cart Name:	Average Impact Force (N)	Impact Time (s)	$Ft = \Delta p$
Cart A	7	0.04	0.28 Ns
Cart B	8	0.03	0.24 Ns
Cart C	2	0.13	0.26 Ns
Cart D	6	0.05	0.3 Ns

8. Which cart experienced the totally **inelastic** collision?
A. Cart A [✓ B. Cart B] C. Cart C D. Cart D E. There's not enough information to tell.

smallest Impulse, so smallest Δp, so smallest ΔV, so smallest V_0

9. All of the carts had the same mass (0.2kg) and closing speed. What was that closing speed?
A. 0.6m/s B. 0.8m/s C. 1.0m/s [✓ D. 1.2m/s] E. Can't tell

$V_{0B} = \frac{0.24kgm/s}{0.2kg} = 1.2m/s$

10. Which of the carts, if any, experienced a perfectly **elastic** collision?
A. Cart A B. Cart B C. Cart C D. Cart D [✓ E. None of the carts]

Perfectly elastic would have same closing and separation speed, so 2X ΔV_B, so $\Delta p = 0.24 \times 2 = 0.48$ Ns

11. An archer floating in the vacuum of space shoots an arrow that sticks into a target that is also floating freely in space. If the "system" includes **only the target and the arrow**, and no other outside forces are applied to the system, we can be sure that…
    a. Momentum and kinetic energy are both conserved.
    b. Neither momentum nor kinetic energy is conserved.
    [✓ c. Momentum is conserved, but kinetic energy is not.]
    d. Momentum is not conserved, but kinetic energy is conserved.
    e. There is no way to tell for sure that any of these is true.

Problems:

1. A 70kg hockey player was skating at a constant speed of 7m/s. Then a 0.3 second long collision slowed her to 3m/s.

   a. (2pts) What was the player's momentum before the collision?

   $\boxed{p_0 = mV_0} = 70kg(7m/s) = \boxed{490 kg m/s}$

   b. (2pts) What force was exerted on the hockey player during the collision?

   $\boxed{Ft = m\Delta V} \Rightarrow F(0.3s) = 70kg(3m/s - 7m/s)$

   $F = \boxed{-933N}$

2. Ned kicked a 0.4kg rock, exerting an average force of 30N. The rock was stationary before Ned kicked it, and Ned's applied force lasted for 0.12seconds.

   a. (2pts) What impulse was applied to the rock?

   $\boxed{Impulse = Ft} \Rightarrow 30N(0.12s) = \boxed{3.6 Ns}$

   b. (2pts) what was the speed of the rock when it left Ned's foot?

   $\boxed{Ft = m\Delta v} \Rightarrow 3.6Ns = 0.4kg(\Delta v)$

   $\Delta v = \boxed{9m/s = v}$

3. A 15kg child riding a 3kg longboard is moving with a velocity of 4m/s (and so is the longboard). When the child jumps off of the longboard, the board continues in moving in the same direction, but it travels at a new velocity of 8m/s. What is the child's velocity just after she jumps off?

   $\boxed{M_{1i}V_{1i} + M_{2i}V_{2i} = M_1 V_1 + M_2 V_2}$

   $15kg(4m/s) + 3kg(4m/s) = 15kg(V_1) + 3kg(8m/s)$

   $72 kgm/s = 15kg(V_1) + 24 kgm/s$

   $\boxed{V_1 = 3,2 m/s}$