Law of Conservation of Energy (for all energy): For any **isolated** (no matter or energy entering or leaving) system, $KE_i + PE_i + OE_i = KE_f + PE_f + OE_f$ OE represents "other energy." Other energy includes any energy that is not mechanical. OE can be chemical, electrostatic, thermal...

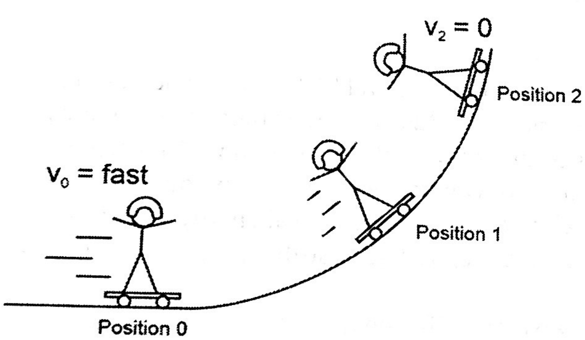
Adding to – or Subtracting From – a system's total Mechanical Energy: Often work is done to add or remove mechanical energy to a system. This work is said to be done by "non-conservative forces," because the total amount of mechanical energy in the system changes – total mechanical energy is *not conserved.* **Work by non-conservative forces is labeled W_{nc}.** A more general equation for mechanical energy takes this work into account...

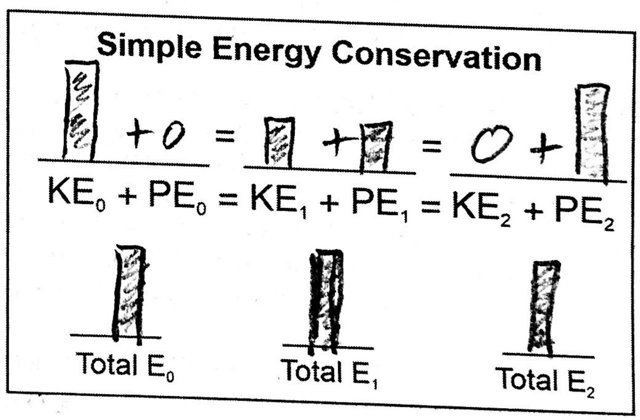
$$KE_{initial} + PE_{initial} + W_{nc} = KE_{final} + PE_{final}.$$

The **Work-Energy Theorem** can be useful, but it can also be tricky to apply, and you can get by without it. If you want to use it, it is technically $W_{net} = \Delta KE$. The *net* amount of work done on an object equals the object's change in KE. *[Here's an example of its trickiness... if you lift a box from the floor and set it on a table, its KE has not changed, so there is no net work done on the box. At first this seems wrong, but it's actually right; you do positive work on the box (non-conservative work, because you're adding PE to the box) and gravity does the same amount of negative work on the box (but this is conservative work, so it does not change the total amount of mechanical energy). The total (net) work is zero.]*

Example – Conservation of Energy with No Non-Conservative Work: A skateboarder is skating up the wall of a half-pipe in a frictionless environment.

Use vertical bars to show how the relative values of the skateboarder's KE and PE, and E_{total} vary at positions 0, 1, and 2.





Is work (a force applied over a distance) done on the skateboarder? Yes

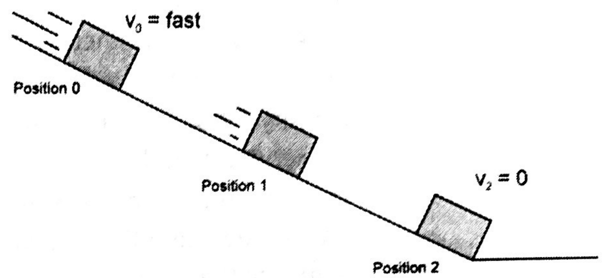
What does the work? Gravity (the Earth)

There is "conservative work" and "non-conservative" work. Conservative work does not change the total mechanical energy of a system, because it is recoverable (e.g., if you compress a spring, you store energy in it, and you can recover that energy by allowing it to push you back). Conservative work allows energy to go in and out of "storage." Non-conservative work <u>does</u> change the total mechanical energy. The energy for non-conservative work cannot be recovered. It cannot be simply moved "in and out of storage."

The work done on this skateboarder -- in this frictionless environment with no external forces -- is __________ (conservative or non-conservative).

Example -- Negative Work by a Non-conservative Force: A box is sliding down a ramp, slowing down at a constant rate until it stops.

- In the top space, use vertical bars to show the relationship between KE, PE, Mechanical Energy, and non-conservative work.
- Identify the source of the non-conservative work.
- In the bottom spaces, use vertical bars to represent the relative values of the system's KE, PE, OE, and E_{total} at various stages in its slide.
- Identify the form of OE in this scenario.



Changes in Mechanical Energy

+ + =

$KE_0 + PE_0 + W_{NC} = KE_1 + PE_1$

Total Mechanical E_0 | Total Mechanical E_1

Negative W_{NC} Removes Mechanical Energy

+ + = 0 + 0

$KE_1 + PE_1 + W_{NC} = KE_2 + PE_2$

0

Total Mechanical E_1 | Total Mechanical E_2

Conservation With All Forms of Energy

Thermal Energy

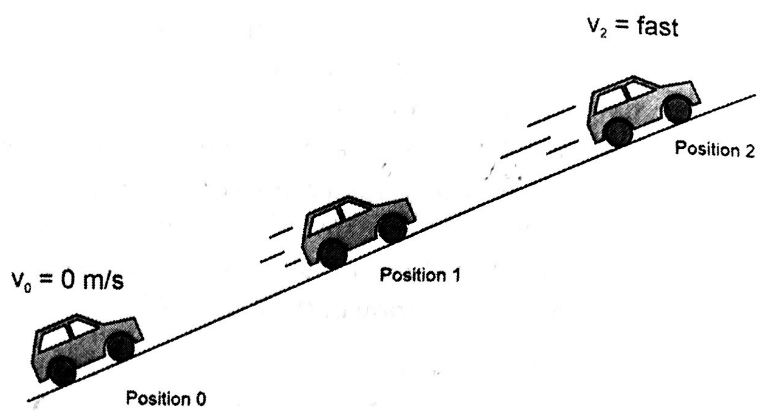
+ + 0 = + + = 0 + 0 +

$KE_0 + PE_0 + OE_0 = KE_1 + PE_1 + OE_1 = KE_2 + PE_2 + OE_2$

Total E_0 | Total E_1 | Total E_2

Example -- Positive Work by a Non-conservative Force: Starting from rest, a car continuously accelerates up a hill.

- In the top space, use vertical bars to show the relationship between KE, PE, Mechanical Energy, and non-conservative work.
- Identify the source of the non-conservative work.
- In the bottom spaces, use vertical bars to represent the relative values of the system's KE, PE, OE, and E_{total} at various stages in its slide.
- Identify the form of OE in this scenario.



Changes in Mechanical Energy

$0 + 0 + \;\; = \;\; + $

Positive W_{NC} adds mechanical Energy

$KE_0 + PE_0 + W_{NC} = KE_1 + PE_1$

0

Total Mechanical E_0

Total Mechanical E_1

$+ \;\; + \;\; = \;\; +$

$KE_1 + PE_1 + W_{NC} = KE_2 + PE_2$

Total Mechanical E_1

Total Mechanical E_2

Conservation With All Forms of Energy

Gas or Electric Charge

$0 + 0 + \;\; = \;\; + \;\; + \;\; = \;\; + \;\; + 0$

$KE_0 + PE_0 + OE_0 = KE_1 + PE_1 + OE_1 = KE_2 + PE_2 + OE_2$

Total E_0

Total E_1

Total E_2

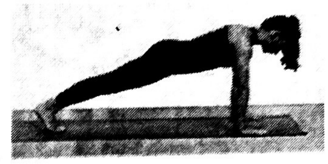
5

Practice Problems:

1. A child pulls a wagon 4m to the right, applying a constant rightward force of 10N. How much work does the child do?

$$W = Fd = 10N(4m) = 40J$$

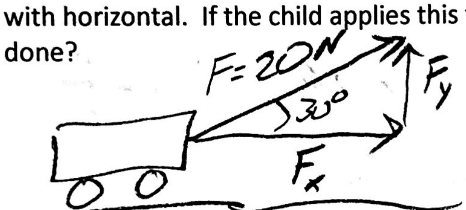
2. A 60kg military cadet holds a plank for 10 seconds. How much work does she do? [Follow the strict physics definition of work]



$$W = Fd = F(0) = 0J$$

Distance is zero, so no work

3. Another child pulls a wagon using a rope. The tension in the rope is 20N, and the rope makes a 30° angle with horizontal. If the child applies this force constantly as the wagon travels 6m, how much work is done?



$$W = Fd = F_x d$$

component of force in direction of displacement

$$F_x = 20N(\cos 30°) = 17.3N$$

$$W = 17.3N(6m) = 104J$$

4. [Energy conservation, PE, KE, W_{nc}] A 0.5kg quadcopter takes off vertically from rest with a constant thrust force of 100N. What is the quadcopter's speed when it reaches a height of 1m? Top

$$PE_0 + KE_0 + W_{NC} = PE + KE$$

$$0J + 0J + F_{thrust}(d) = mgh + \tfrac{1}{2}mv^2$$

Bottom

$$0J + 0J + 100N(1m) = 0.5kg(9.8m/s^2)(1m) + 0.5(0.5kg)v^2$$

$$100J = 4.9J + 0.25v^2$$

$$v = 19.5m/s$$

6

5. [Energy Conservation, Work, Power] A 60kg student climbs 12m up a vertical rock wall in 50 seconds. The student's speed is constant. → let's use $1\,m/s$

a. Approximately how much work did the student do?

$$PE_0 + KE_0 + W_{NC} = PE + KE$$

$$0 + \tfrac{1}{2}(60kg)(1m/s)^2 + W_{NC} = 60kg(9.8m/s^2)(12m) + \tfrac{1}{2}(60kg)(1m/s)^2$$

$$30J + W_{NC} = 7056J + 30J$$

$$W_{NC} = 7{,}056J$$

we could have canceled these in the beginning

b. What was the student's average power output, in Watts?

$$P = \frac{W}{t} = \frac{7056J}{50s} = 141W$$

c. How long would the climb have taken if the student's power output had been 1 horsepower?

$$1hp = 746W \qquad P = \frac{W}{t} \qquad 746W = \frac{7056J}{t} \Rightarrow t = 9.46s$$

d*. Just for fun, contemplate the net amount of work done on the student.

$$W_{Net} = F_{net}d$$

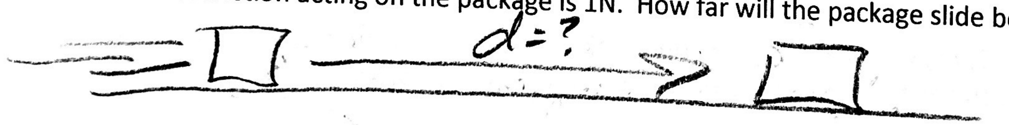
$$\Sigma F = ma = m(0) = 0 \qquad W_{net} = 0(d) = 0$$

Wall does positive work on student

Gravity does negative work on student

$W_{net} = 0$

6. [Energy conservation, KE, W_{nc}] A 2kg package is sliding across a level surface at a velocity of 3m/s. The force of friction acting on the package is 1N. How far will the package slide before it stops?



$$PE_0 + KE_0 + W_{NC} = PE + KE$$

$$0 + \tfrac{1}{2}mv^2 + F_{Fr}(d) = 0 + 0$$

$$\tfrac{1}{2}(2kg)(3m/s)^2 - 1N(d) = 0$$

$$9J = d(1N)$$

$$d = 9m$$

\# 8 + 7 are reversed here ↓

7

8 [Energy conservation, PE, KE, W_{nc}] A 3kg watermelon is dropped from rest at a height of 100m. It lands on a large spring, contacting the spring at a height of 15m and then compressing the spring a distance of 5m before coming to rest on the compressed spring at a height of 10m.

a. What is its velocity at the moment it first hits the spring? 40.8 m/s

b. How much energy is stored in the spring once the watermelon comes to rest? 2646 J

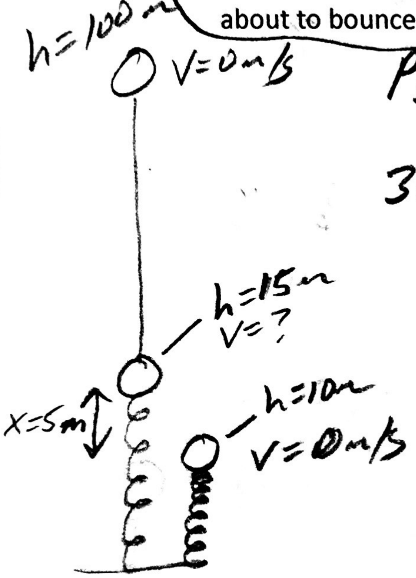
c. What is the spring constant? $PE_s = \frac{1}{2}kx^2 = \frac{1}{2}k(5m)^2 = 2646J$

$k = 212\ N/m$

d. How much force is the spring exerting on the watermelon when the watermelon comes to rest [It's about to bounce back.]

$F_{sp.} = kx = 212 N/m (5m)$

$F_s = 1{,}060N$



$$PE_{G_{100m}} + KE_{100m} + W_{NC} = PE_{G_{15m}} + KE_{15m}$$

$$3kg(9.8m/s^2)(100m) + 0 + 0 = 3kg(9.8m/s^2)(15m) + \frac{1}{2}(3kg)v^2$$

$$2940J = 441J + 1.5(v^2)$$

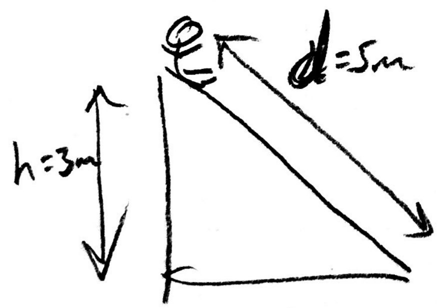
$$v = 40.8 m/s$$

$$PE_{100m} + KE_{100m} = PE_{10m} + KE_{10m}$$

$$2940J + 0 = PE_{g_{10m}} + PE_{s_{10m}} + 0$$

$$2940J = 3kg(9.8m/s^2)(10m) + PE_s \Rightarrow PE_s = 2646J$$

7 [Energy Conservation, PE, KE, W_{nc}] A 20kg child sits at rest at the top of a slide which is 5m long and 3m high. As the child slides down the slide, the child experiences a constant 5N force of friction. What is the child's speed upon reaching the bottom of the slide?



$$PE_{Top} + KE_{Top} + W_{NC} = PE_{Bottom} + KE_{Bottom}$$

$$mgh + 0 - F_{Fr}d = 0 + \frac{1}{2}mv^2$$

$$20kg(9.8m/s^2)3m - (5N)(5m) = \frac{1}{2}(20kg)v^2$$

$$588J - 25J = 10kg(v^2)$$

$$v = 7.5 m/s$$

8

Location	Height (m)	Velocity (m/s)	Gravitational Potential Energy (J)	Spring Potential Energy (J)	Kinetic Energy (J)	Total Mechanical Energy (J)	Electrical Potential Energy (J)	Thermal Energy (J)	Total Energy (J)
A	50	0	98,000	0	0	98,000	100,000	0	198,000
B	90m	0	176,400	0	0	176,400	21,600	0	198,000
C	60	24.2	117,600	0	58,800	176,400	21,600	0	198,000
D	55	26.19	107,800	0	68,600	176,400	21,600	0	198k
E	55	19.7	107,800	0	48,600	156,400	21,600	20,000	198k
F	40	0	78,400	78,000	0	156,400	21,600	20,000	198k

1.

a. $\boxed{W = Fd} = 70N(4m) = \boxed{280J}$

b. $\boxed{P = \frac{W}{\Delta t}} = \frac{280J}{6s} = \boxed{46.7W}$

c. $\boxed{KE = \frac{1}{2}mv^2} = \frac{1}{2}(10kg)(3m/s)^2$

$\boxed{KE = 45J}$

d. $\boxed{PE = mgh} = 10kg(9.8m/s^2)(1.5m)$

$\boxed{PE = 147J}$

e. $\boxed{KE_0 + PE_0 + W_{NC} = KE + PE}$

work by external forces or friction — Friction work and goat work

$0 + 0 + W_{NC} = 45J + 147J$

$W_{NC} = 192J$

$W_{NC} = \text{Goat work} + \text{Friction work}$

$192J = 280J + W_{Friction}$

$\boxed{W_{Friction} = -88J}$

f. $\boxed{W_{Friction} = Fd} \Rightarrow -88J = F_{Fr}(4m)$

$\boxed{F_{Fr} = -22N}$

g. $\boxed{W = F_{\parallel}d} = \cos 25(70N)(4m) = \boxed{254J}$

9

ems: **Correct Units are required for full credit! Show starting equations and work for partial credit.

10

1. How much work is done on a box that is pushed 6m to the right by a constant rightward net force of 10N?

$\boxed{W = Fd}$ $W = 10N(6m) = \boxed{60J}$ why Nm??

2. A sled cat pulls its load by applying a constant 70N tension force in the direction shown in the diagram (20° above horizontal). If the sled cat pulls its sled 10,000m horizontally (parallel to the dotted line), how much work is does the cat do on the sled?

$\boxed{W = F_{\parallel} d}$ 700000 = (-1)

$F_{\parallel} = \cos 20(70N) = 65.8N$

$W = 65.8N(10,000m) = \boxed{658,000J} = 6.58\times10^{5}J$

285,657 in Rad.

3. Suppose a 450kg racehorse is initially at rest. For exactly 6 seconds, the horse generates a constant power of 15,000W (about 20.1 horsepower, surprisingly) and uses all of that power to accelerate. The horse travels across level ground.

Form (1) Subst (1/2) math (1/2)

-1/2 a. How much work does the racehorse do during this 6 second period?

$\boxed{P = \frac{W}{t}}$ $15,000W = \frac{W}{6s}$ $\Rightarrow$ $\boxed{W = 90,000J}$

b. Assuming that none of this work is lost to "other energy," what is the kinetic energy of the horse after 6 seconds?

$PE_0 + KE_0 + W_{NC} = PE + KE$ $\boxed{W_{NC} = KE}$ $\boxed{90,000J}$

0 0 Worth 1/2 0

d. What is the horse's speed after 6 seconds?

$\boxed{KE = \frac{1}{2}mv^2}$

$90,000J = \frac{1}{2}(450kg)v^2$

$\boxed{v = 20m/s}$

10/10

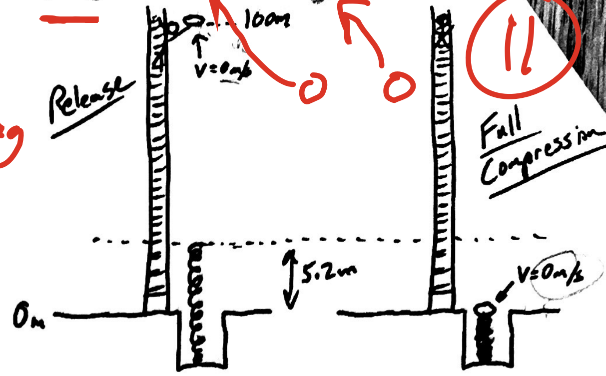
7.35

24.5

$0 \to PE_{0S} + PE_{0G} + KE_0 + W_{NC} = PE_S + KE + PE_G$

0 Drag 0 0 11

4. Ned thinks he has found a clever method of compressing his large spring. He carries a 6kg watermelon to a height of 100m and drops it on the spring. The watermelon falls until it hits the spring and comes to rest, compressing the spring a distance of 5.2m and stopping at ground level. The spring's k = 350N/m.



a. How much energy is stored in Ned's spring at the moment when the watermelon's downward motion ceases?

$\boxed{PE_S = \frac{1}{2}kx^2} = \frac{1}{2}\,350\,N/m\,(5.2m)^2 = \boxed{4732J}$

b. If we assume that all of the non-conservative work done on the watermelon is done by air resistance (drag) acting over a distance of 100m, what average drag force is exerted on the watermelon during its fall?

58.8 = -1

$\boxed{mgh + W_{NC} = PE_S}$

-3/4

$6kg(9.8m/s^2)(100m) + F_{Drag}(100m) = 4732J$

$5880J + F_{drag}(100m) = 4732J$

$F_{drag}(100m) = -1,148J$

$\boxed{F_{drag} = -11.5N}$

12.11 N ok

c. What is the % efficiency of Ned's method of compressing his spring.

$\boxed{Eff. = \frac{Output}{Input}(100\%)} = \frac{4732J}{5880J}(100\%) = \boxed{80.5\%}$

Formula only $-1\frac{1}{2}$

5. A 0.15kg graduation cap is tossed directly upward at a graduation ceremony (in a vacuum, on Earth's surface). The cap is released from the thrower's hand when it is 2m above the ground. At that point it is moving upward with 6J of kinetic energy.

a. How much PE does the graduation cap have at the moment when it leaves the hand? (at h=2m)

$\boxed{PE_g = mgh} = 0.15kg(9.8m/s^2)(2m) = \boxed{2.94J}$

b. How much PE does the graduation cap have when it reaches its maximum height?

2m

$\boxed{PE + KE = PE}$ top

$2.94J + 6J = PE_{top}$

$\boxed{PE_{top} = 8.94J}$

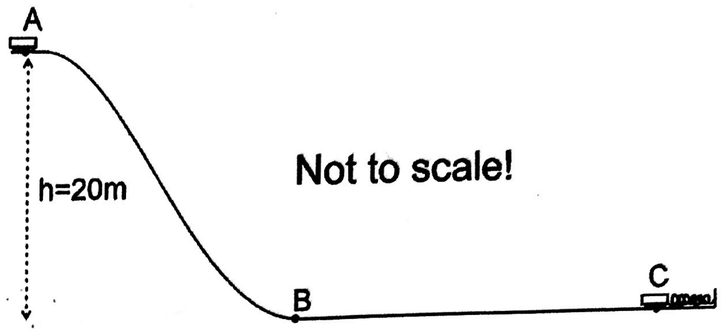
c. How much kinetic energy will the graduation cap have just before it hits the ground?

$PE_{top} = KE_{ground} = \boxed{8.94J}$

12/12

Starting from rest, a **600kg** roller coaster leaves point A and travels frictionlessly down a ramp to point B. At point B, the coaster travels horizontally while its brakes apply a **-2,500 N** force of friction to slow it down. As friction continues to slow the coaster, the coaster contacts a huge spring (**k=10,000N/m**), finally coming to stop at point C, after compressing the spring a distance of **3m.** When the coaster comes to a stop, the spring pushes it back again.

12



A. Find the coaster's PE at point A.

$\boxed{PE = mgh}$

$= 600kg(9.8m/s^2)(20m)$

$\boxed{117,600J}$

B. Find the coaster's KE at point B.

$PE_{Top} = KE_{Bot} = \boxed{117,600J}$

$PE_0 + KE_0 = PE + KE$

0 0

C. Find the PE stored in the spring, at the moment the coaster comes to rest at point C.

$\boxed{PE_{Sp} = \frac{1}{2}kx^2} = \frac{1}{2}(10,000 N/m)(3m)^2$

$= \boxed{45,000J}$

D. Between points B and C the coaster experienced a constant 2,500N force of friction from its brakes. What is the distance from B to C?

$\boxed{KE_B + W_{NC} = PE_{SC}}$

47.05m No pts for units close

-1

$117,600J - 2,500N(d) = 45,000J$

$2,500(d) = 72,700 \Rightarrow \boxed{d = 29.0m}$

E. How much force does the spring exert on the coaster at the moment that the coaster comes to rest (before bouncing back again) at point C?

$\boxed{F_s = kx} = 10,000 N/m (3m) = \boxed{30,000N}$

15000

w=Fd (-1)

10/10