

$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

1. Calculate the gravitational force between a 1,200kg car and a 60kg person when their centers of mass are 5m apart.

$$F_g = G \frac{M_1 M_2}{r^2} = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \left(\frac{(1,200 \text{ kg})(60 \text{ kg})}{(5 \text{ m})^2} \right)$$

$F_g = 1.92 \times 10^{-7} \text{ N}$

2. A carnivalgoer who weighs 600N passes through a loop-the-loop at a constant speed of 20m/s. Even though they are upside-down, at the top of the loop they feel heavier than normal – as if they weigh 900N. What is the radius of the loop?



$$\Sigma F = -\frac{mv^2}{r}$$

$$\Sigma F = -F_N - mg$$

$$-\frac{mv^2}{r} = -F_N - mg \Rightarrow \frac{mv^2}{r} = F_N - mg$$

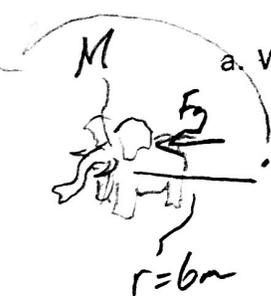
$61.2 \text{ kg} (20 \text{ m/s})^2 / r = 900 \text{ N} - 600 \text{ N}$

600N = weight
900N = sensation of weight

$W = m(9.8 \text{ m/s}^2) = 600 \text{ N} \Rightarrow m = 61.2 \text{ kg}$

$r = 16.3 \text{ m}$

3. A 10,000kg bush elephant is floating in empty space, far from any other object. A tiny speck of dust is orbiting the elephant in a circular orbit at an orbital radius of 6m.



- a. What is the velocity of the dust speck?

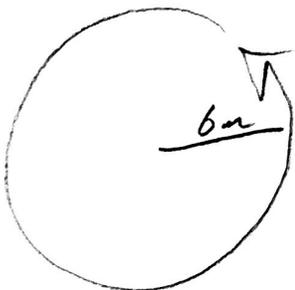
$$\Sigma F = \frac{mv^2}{r}$$

$$\Sigma F = G \frac{Mm}{r^2}$$

$$\frac{mv^2}{r} = G \frac{Mm}{r^2} \Rightarrow v^2 = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \frac{(10,000 \text{ kg})}{(6 \text{ m})}$$

$v = 3.33 \times 10^{-4} \text{ m/s}$

- b. How long will it take the dust speck to orbit the elephant once?



distance = $2(\pi)r$

$$t = \frac{d}{v} = \frac{2(\pi)6 \text{ m}}{3.33 \times 10^{-4} \text{ m/s}}$$

$1.13 \times 10^5 \text{ s}$

$= 1.33 \text{ days}$

Problem Changed

4. Earth's mass is 5.97×10^{24} kg. Its radius is 6.37×10^6 m. How high above the Earth's surface would you need to be in order to free-fall with an acceleration of $g = 4 \text{ m/s}^2$?

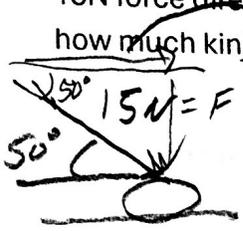
$$mg = G \frac{Mm}{r^2} \Rightarrow g = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \left(\frac{5.97 \times 10^{24} \text{ kg}}{(6.39 \times 10^6 \text{ m})^2} \right)$$

orbital $r = 9.78 \times 10^6 \text{ m}$

$g = 9.75 \text{ m/s}^2$

Altitude = orbital radius - Earth radius = $3.6 \times 10^6 \text{ m}$

5. A lazy curler uses their foot to accelerate a curling stone. They push the 20kg stone with a 15N force directed at a 50 degree downward angle (relative to horizontal). Ignoring friction, how much kinetic energy does the stone have after they push it a horizontal distance of 2m?



$$F_x = 15 \text{ N} (\cos 50^\circ) = 9.64 \text{ N}$$

$$W = 9.64 \text{ N} (2 \text{ m}) = 19.3 \text{ J}$$



- *6. Pat has a machine, and its purpose is to lift hay bales. The machine uses 1500W of power. Over a time interval of 11 seconds, the machine lifts a 25kg hay bale from a height of ~~15m~~ ^{12m} height of 15m. During this time, the hay bale moves at a constant speed.

- a. How much energy does the machine use during these 10 seconds?

$$P = \frac{W}{t} = \frac{E}{t} \quad 1500 \text{ W} = \frac{1500 \text{ J}}{\text{s}} = \frac{E}{10 \text{ s}} \Rightarrow E = 15,000 \text{ J}$$

- b. How much work does the machine do on the hay bale?

~~$PE_0 + KE_0 + W_{nc} = PE + KE$~~ Constant speed, so these cancel

$$0 + W_{nc} = mgh$$

$$W_{nc} = 25 \text{ kg} (9.8 \text{ m/s}^2) (12 \text{ m}) = 2940 \text{ J}$$

- c. What is the machine's % efficiency?

$$\% \text{ Eff} = \frac{\text{Output}}{\text{Input}} (100\%) = \frac{2940 \text{ J}}{15,000 \text{ J}} = 19.6\%$$

7. 16 students pull ropes, causing a sled to accelerate from rest on level ground. For every student pulling, a propulsive force of 24N gets applied to the sled. This force is applied to the sled over a distance of 17.4m. The total mass of the sled and the occupant is 83kg, and the coefficient of friction between the sled and the snow is 0.12.

a. How much work do the students do on the sled?

$$W = Fd = 16 \text{ students} \left(\frac{24 \text{ N}}{\text{student}} \right) (17.4 \text{ m}) = 6,682 \text{ J}$$

b. How much work does friction do on the sled?

$$W = Fd \rightarrow W = -0.12(83 \text{ kg})(9.8 \text{ m/s}^2)(17.4 \text{ m})$$

$$\frac{F_f}{F_N} = \mu \frac{F_N}{F_N} = \mu mg \rightarrow W = -1,698 \text{ J}$$

c. What is the sled's velocity after it accelerates for the full 17.4m?

$$PE_0 + KE_0 + W_f + W_{\text{pull}} = PE + KE$$

$$0 + 0 - 1,698 \text{ J} + 6,682 \text{ J} = 0 + \frac{1}{2}(83 \text{ kg})v^2$$

$$v = 11.0 \text{ m/s}$$

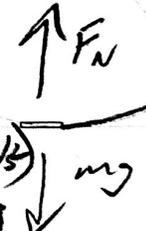
8. Next the sled from the previous question begins to climb up a ramp. The entry to the ramp is an arc of a circle with a radius of 9m. What value of g is felt by the sled and its occupant once they have entered this arc?

$$\Sigma F = \frac{mv^2}{r} \Rightarrow \frac{mv^2}{r} = F_N - mg$$

$$\Sigma F = F_N - mg \quad F_N = m \left(\frac{v^2}{r} + g \right)$$

$$F_N = 83 \text{ kg} \left(\frac{(11 \text{ m/s})^2}{9 \text{ m}} + 9.8 \text{ m/s}^2 \right) = 1929 \text{ N}$$

g's felt = $\frac{F_N}{W} = \frac{1929 \text{ N}}{83 \text{ kg}(9.8 \text{ m/s}^2)} = 2.37g$



9. The same sled continues to travel up the ramp to a height of 0.8m before flying into the air. If we ignore friction and drag for this event, what is the sled's velocity when it reaches the top of the ramp?

$$PE_0 + KE_0 = PE + KE$$

$$0 + \frac{1}{2}mv_0^2 = mgh + \frac{1}{2}mv^2$$

$$\frac{1}{2}(11 \text{ m/s})^2 = 9.8 \text{ m/s}^2(0.8 \text{ m}) + \frac{1}{2}v^2$$

$$v = 10.3 \text{ m/s}$$

10. A spring is compressed a distance of 0.4m, and it is placed next to a 0.15kg puck. The puck and the spring are at a height of 5m. All of the energy stored in the spring is then used to launch the puck rightward, with no friction or drag. When the puck leaves the spring, it is traveling horizontally at a speed of 8m/s. The puck slides frictionlessly up a hill until its speed has slowed to 4m/s. It then slides down the other side of the hill and encounters a section of track where it experiences a constant friction force of 4N. The puck continues to experience this friction until it returns to its original height of 5m, where it comes to rest.

b. How much gravitational potential energy does the puck have in the beginning, when it is at a height of 5m?

~~$$PE_g = mgh = 0.15 \text{ kg} (9.8 \text{ m/s}^2) (5 \text{ m}) = 7.35 \text{ J}$$~~

- a. How much energy was stored in the spring before it pushed the puck?

$$PE_s + KE_s = PE + KE$$

$$PE_s + 0 = 0 + \frac{1}{2}mv^2$$

$$PE_s = \frac{1}{2}(0.15 \text{ kg})(8 \text{ m/s})^2$$

$$PE_s = 4.8 \text{ J}$$

- k. What is the spring's constant, k?

$$PE_s = \frac{1}{2}kx^2 \Rightarrow \frac{1}{2}k(0.4 \text{ m})^2 = 4.8 \text{ J} \Rightarrow k = 56 \frac{\text{N}}{\text{m}}$$

- c. What maximum force does the spring apply?

$$F_s = kx = 56 \frac{\text{N}}{\text{m}} (0.4 \text{ m}) = 22.4 \text{ N}$$

- d. What is the height of the top of the hill?

After spring at 5m

$$PE_s + KE_s = PE + KE$$

$$0 + 4.8 \text{ J} = 0.15 \text{ kg} (9.8 \text{ m/s}^2) (h) + \frac{1}{2} (0.15 \text{ kg}) (4 \text{ m/s})^2$$

- e. How much work is done on the puck by friction?

$$PE_s + KE_s + W_{fr} = PE + KE$$

$$0 + 4.8 \text{ J} + W_{fr} = 0 + 0 \Rightarrow W_{fr} = -4.8 \text{ J}$$

$$h = 2.45 \text{ m}$$

- f. How far does the puck slide while it experiences friction?

$$W_{fr} = F_{fr} d \Rightarrow -4.8 \text{ J} = -4 \text{ N} (d)$$

$$d = 1.2 \text{ m}$$